LEARNING MATERIAL

SEMESTER : **4TH SEMESTER**

BRANCH : **MECHANICAL ENGINEERING**

THEORY SUBJECT : **THEORY OF MACHINES (TH – 1)**

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Books

- Thomas Bevan
- Ghos & Malik
- Rattan (Amitabh singh

Mechanics

- H.C. Verma
- Beer & Johnson
- Library . genetic

SOM

- Gere & Timoshenko
- Beer Hibbeler
- G.H. Ryder - [ES]

(M)

- Vicky Sthevex

T. O. M.

[10 - 12 Marks]

- Beer & Johnson (Vector Mechanics)
- J. L. Meriam (Dynamics
- Hibbler
- Grover Mech. Vib.
- V.P. Singh - Mech. Vib

Power by :-

[illegible]

Theory of M/c

- Kinematic of M/c
- Dynamic of M/c

Engg Mechanics

- Static
- Dynamics
  - Kinetic
  - Kinematic

- In Kinematics of Machines : We do the displacement, velocity & Acceleration analysis of different components of the m/c.
- We are not concerned with external forces acting on it
- In dynamics of Machine we are concerned with all the forces that is external force, restoring force, damping force etc. acting on m/c

⇒ M/c :

Cell → Tissue → Organs → Organ system → Human body

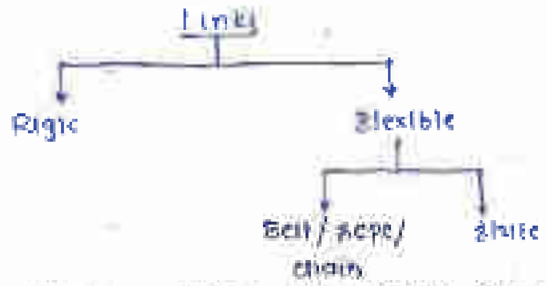
Link / Element → pair / Joint → chain / linkage → Mechanism → Machine

Link / Element → It is smallest unit of any machine

→ A link should be a resistant body (i.e. deformable) it need not to be rigid body always.

→ Link must be able to transfer the relative motion.

⇒ **Types of link** :-



UPSC : All the flexible links are having one directional rigidity that is they will work under a specific condition only.

Ex Belt / Rope will work as link when it is subjected to tension

Where fluid will work as link when it is subjected to compression

→ Spring follows Hooke's law

$F_s \propto -x$

is used for exerting force (restoring force)

UPSC :

→ Springs are mainly used to exert the restoring force therefore we can not considered spring as kinematic link

→ Several parts manufactured separately but does not have relative motion between them will be considered as one link.

Ex Crank, crankshaft, flywheel, Driven flange of clutch form one link only because all have same speed ω.

2

Pair / Joint → The inter connection between two or more links in such a manner that it permits the desired relative motion to get transferred will be known as kinematic pair.

① On the basis of degree of freedom
② On the basis of types of contact
③ On the basis of type of closure
④ On the basis of no. of links to be connected.

✱ Degree of freedom (D.O.F)

— Total no. of independent co-ordinates that is fully variable required to define the motion completely is known as degree of freedom

Translation (S)	Rotation (θ)
S_x	$θ_x$
S_y	$θ_y$
S_z	$θ_z$

3T + 3R = 6

max. dof = 6 | in 3D   is '6' not less " 12"

In 2D :

planar :

Translation	Rotation
S_x	$θ_y$
S_z	

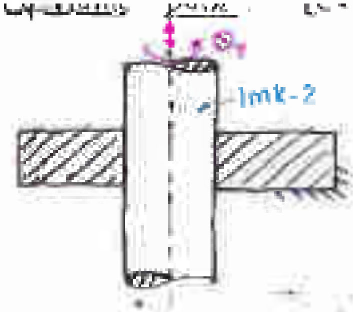
? absorbed / Restricted / constrained

T	R
S_y	$θ_x$
	$θ_z$

max dof = 3 | in 2D

Actual dof = max. possible dof − Restricted dof

i) Cylindrical pair [C - pair]



Possible dof

T	R
S_y	θ_y

Arrested dof

S_x	θ_x
S_z	θ_z

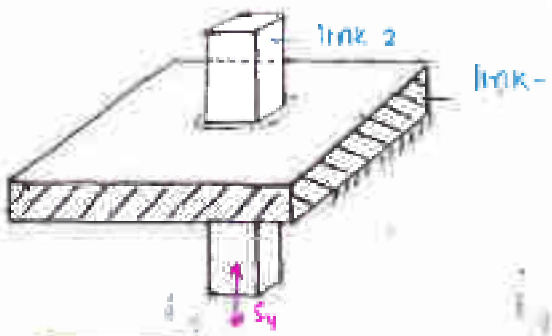
Act - dof = max possible dof - arrested dof

= 6 - 4

dof = 2

Ex. shaft in bearing

ii) Prismatic pair : [P - Pair]



Bending moment is parabolic half circle (M)

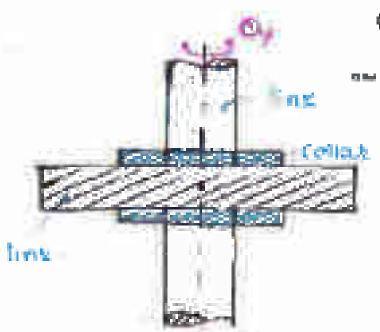
Torsional moment is ellipse shape (T)

— If beam is hinged at both end then at both ends are bending moment is zero i.e. ΣM = 0

dof = 1

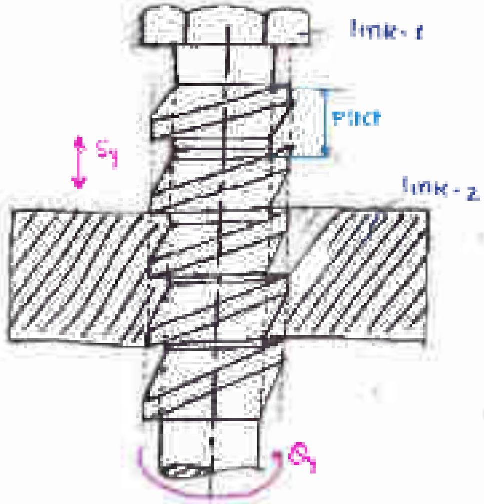
Ex. shaft in between piston - cylinder, crank + slotted bar.

iii) Revolute pair [R - pair] (Lower pair)



Ex. Thrust bearing

dof = 1



$\Delta s_y = f(\Delta\theta_y)$

D.P.V I.P.V

$$\frac{\Delta s_y}{lead} = \frac{\Delta\theta_y}{2\pi}$$ GATE (5M)

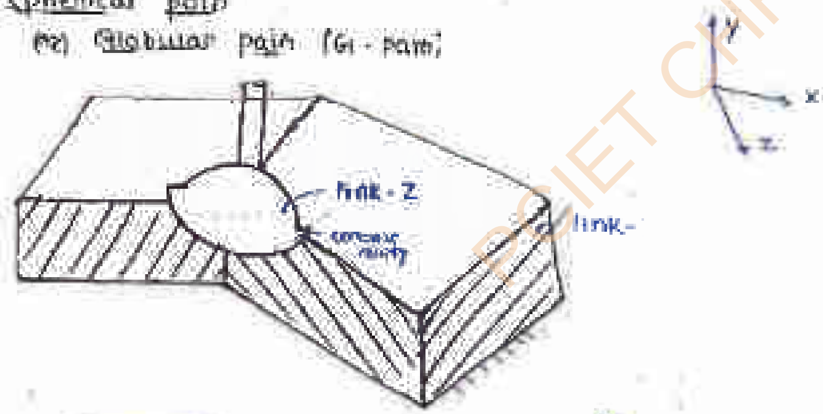
Ex: Nut & screw
power screw

Lead - The axial dist^n travelled by nut in complete rotation

pitch - Distance between two similar points on successive threads measured parallel to the pitch centroidal axis

- Lead = pitch ⇒ single start thread
- Lead = 2 × p ⇒ Double start thread

v) Spherical pair
(or) Globular pair (G - pair)



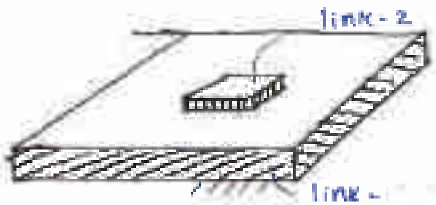
dof = 3

α_x
α_y
α_z

Ex: pendulum in common axis

Ex: Ball & socket joint
Joystick

vi) Planar pair ...



d.o.f = 3

Ex: cube on surface

(2) On the basis of type of contact :

Kinematic pair

(1) Lower pair

- If there is area contact between the moving elements it is known as lower pair.

Ex: All above ex. are area contact

(2) Higher pair

- If there is point or line contact between the moving element it comes under Higher pair.

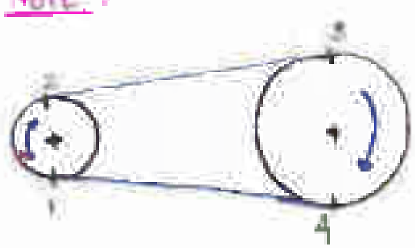
Kinematic Pair

- Lower pair
  - Linear motion pair [d.o.f = 1]
    Ex: Revolute, prismatic
  - Surface motion pair [d.o.f > 1]
    Ex: cylindrical, spherical
- Higher pair
  - Wrapping pair

- If one link is wrapped over another link it is known as wrapping pair [d.o.f = 2]

Ex: Belt & pulley

Note :



- At every entry & exit to pulley system is doing rolling with slipping ; Hence it is example of Higher pair.

[Total No. Higher pair = 4]

| Rolling = Translation + Rotation |

→ In plane total 3 = 2 T + 1 R

→ Higher pair always restricted 1 D.O.F

→ All the surface motion pairs will violate Reuleaux def.

for Lower pair

case-(a) link-1 is stationary
link-2 is moving
(pure translation)

P_1 - - - - - - - - -

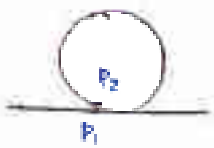
| K_{P_1} = st. line |

case-(b) link-2 is stationary
link-1 is moving
(pure translation)

P_2 - - - - - - - - -

| K_{P_2} = st. line |

for Higher pair



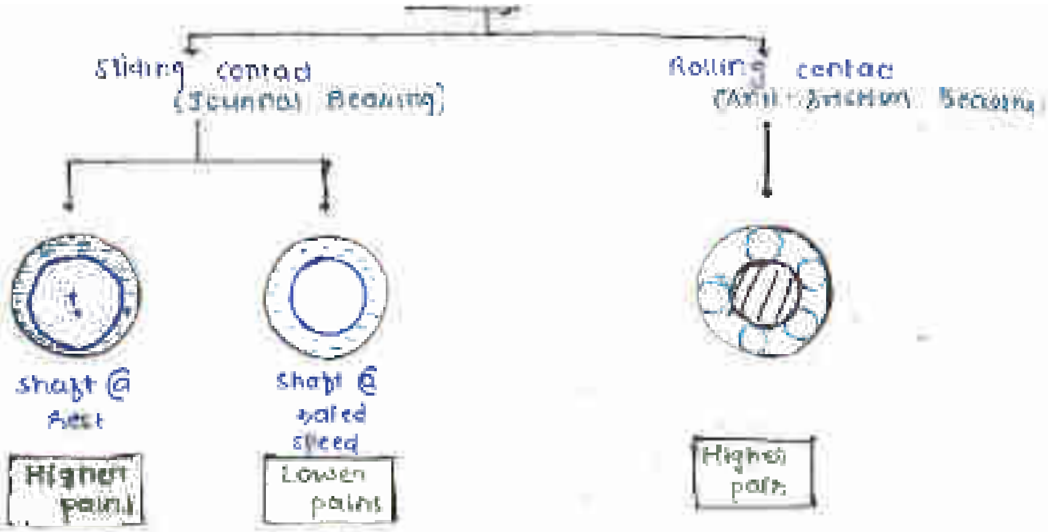
case-(a) :- link-1 (st. line) is fixed
link-2 (circle) is moving
(pure rolling)

| K_{P_2} = cycloid = K_{P_2} |

case-(b) :- link-1 (circle) is fixed
link-2 (st. line) is moving
(pure rolling)

| K_{P_1} = involute = K_{P_1} |

Lower pairs	Higher pairs
1) Full joint	Half joint
2) Area contact	point / Line contact
3) can be inverted	can not be inverted
4) More friction	Less friction
5) Req. more lubrication	Less lubrication
6) There will be more wear & tear due to friction	Higher pairs are subjected to more wear & tear under same max. load
7) Lower pair can efficiently carry more load ...	Higher pair require less lubrication
	□ cam & follower



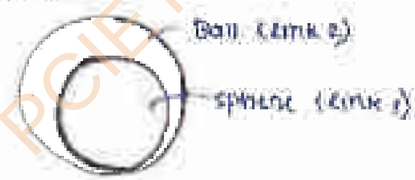
Note: Bearings are not kinematic pair.

- Bearings are mainly used to hold shaft in correct position & bear the load, it has nothing to do the transfer of relative motion. Hence Bearings are not kinematic pairs they are only pairs.

(3) On the Basis of type of closure

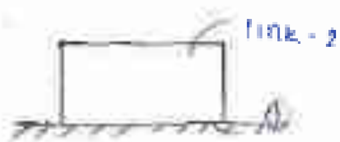
(i) Closed pair

- If the link is completely enclosed in to another link
- The link which is inside another can not bring out without failure of external link



(ii) Unclosed pair or open pair

- If pair is open in space

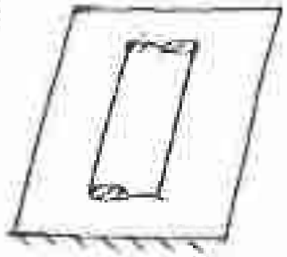


(ii) Ex

T	R
S_x	θ_x
S_z	θ_y
	θ_z

dof = 5

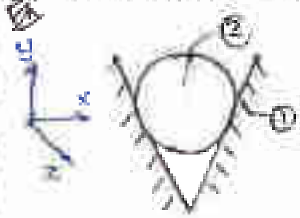
(iii) Slab on a surface :
Ex



T	R
S_x	θ_y
S_z	θ_z

dof = 4

(iv) Cylindrical Bar in V-groove : (GATE)
Ex



Restricted

T	R
S_x	θ_x
S_y	θ_y

Restricted dof = 4
dof = 2

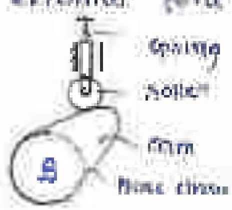
⇒ (*)

(i) Form closed pair
- If the contact that is formed between mating element due to geometrical specification the pair is known as formed closed pair.

Ex
shaft & key
Nut & screw.

(ii) Force closed pair
- If the contact between mating element is due to some force either self wt of link or some external force (spring force) known as forced closed pair.

Ex cam & follower
(Higher pair)



- Every link have minimum one or two nodes.
- If two links are connected at one node it is known as binary joint.

[: Binary Joint]

- If three links are connected at one node is known as ternary joint.

[Ternary Joint :

1 T Joint = 1 B + 1 B

1 T Joint = 2 B Joint

- If four links are connected

1 Q Joint = 3 B Joint

Note

On the basis of motion between links the kinematic pairs are classified into 3 - categories.

1) Completely constrained pairs

- If the motion between links is in unique direction and of unique type and does not depend on direction of force applied it is an example of completely constrained pairs.

Ex: prismatic pair, (P pair)

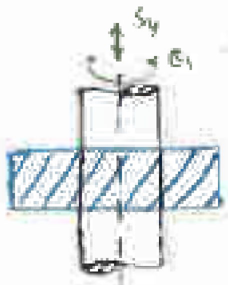
2) Incompletely constrained pairs

- If the motion is possible in more than one direction or more than one type it is known as incompleted constrained pairs.

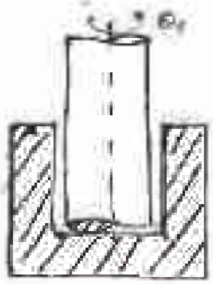
Ex: Cylindrical pair (C pair)

- If an incompletely constrained pair is converted into completely constrained pair either by applying some force or by changing the geometry of specification of mating element known as successfully constrained pair.

Ex - Rollers bearing (exclude balls)
- piston - cylinder



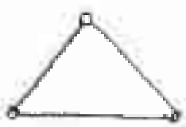
shaft in radial bearing



foot step bearing

3 → Kinematic chain

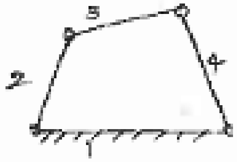
conditions :
1) 1st link should be connected to the last link directly or indirectly
2) It should able to transfer desired relative motion.



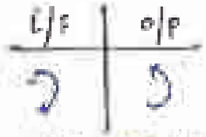
3 bar closed chain

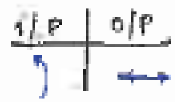
4 bar closed chain

Mechanism :- If one link of chain is fixed and it is able to either transfer / transform f (or) both to the relative motion it is known as mechanism.



four bar mechanism





Newton's 2nd law
- Rate of change of momentum $F = \frac{d(mv)}{dt}$

5
→ **Machine**

- It is combination of various links and parts in such a manner that it is able to transfer / transform or both to the motion, force or power from some source to the load.

Ex I.C engine
Lathe m/c
Robots

Mechanism	vs Machine
- A mechanism is simple model for an complex machine. It analogies to FBD (Used for analysis in SOM - FBD Thermal - system)	- Machine consist of several mechanism. Hence we can say every machine considered as mechanism but every mechanism need not be m/c always.
- Several mechanism combined together may result in m/c	
- Ex clock (is not transform any energy only motion transform) Type writer	

- 1 link is having 3 d.o.f (in planar chain or mechanism)
- If there are 'n' no. of links.

Total no. of dof = 3n (for 'n' no. of links)

- Let us suppose there are j lower pairs (linear motion) [dof = 1]
  - ↳ equivalent no. of binary joints

→ Restricted d.o.f. due to linear motion pair = 2 (binary joints)

total restricted dof = 2j

Actual dof = max^m possible dof − max^m restricted dof

$$dof = 3n - 2j$$

⇒ Effect of Higher pairs

- each higher pair restricted one dof. Let there is 'h' no of higher pairs
- Hence dof restricted by 'h' higher pairs = h

$$dof = 3n - 2j - h$$ ⇐ Chain

In Mechanism

In mechanism one link is fixed / ground / frame

$$dof = 3n - 2j - h - 3$$

$$dof = 3(n-1) - 2j - h$$ ⇐ for Mechanism

It is "Kutzback Equation".

dof < 0 → Super structure / indeterminate structure.

dof = 0 → structure / frame / Truss

dof ≥ 1
- d.o.f = 1 ⇒ Kinematic / Constrained Mechanism
- d.o.f > 1 ⇒ Unconstrained Mechanism

→ Physical Interpretation d.o.f :-

- Degree of freedom predicts No. of para variable or No. of equations required between input & output motion
- Degree of freedom predicts about the No. of links that should be controlled by input (or No. of para variables that should be controlled) in order to have constrained mechanism

Grubler's criterion

dof = 1
h = 0
← constrained mechanism

$\therefore 3(n-1) - 2j - 0 = 1$

$3n - 3 - 2j = 1$

$3n = 2j + 4$

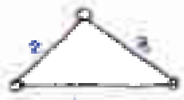
$n_{min} = 4$

$3n$ = even no
$2j$ = even
4 = even

$n = 1$ ×
$n = 2$ ×
$n = 3$ (odd)
$n = 4$ ✓

- According grubler's criteria to make a mechanism consist of all the lower pair having dof = 1 is actual 4.

Ex. [1]



$f = 3(n-1) - 2j - h$
$= 3(3-1) - 2(3)$
$= 0$

- It is chain (not a mechanism) is one link is fixed then become frame.

$n = 3$
$j = 3$
$h = 0$

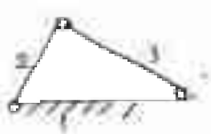
$dof = 3n - 2j - h$ (chain)
$= 9 - 6$
$= 3$

Ex. [2]

$f = 3(4) - 2(4)$
$f = 4$

$n = 4$
$j = 4$
$h = 0$

[3]



- Frame one link is fixed so it mechanism then eq^n

$dof = 3(n-1) - 2j - h$

$dof = 3(3-1) - 2(3) - 0$

$dof = 0$

$n = 3$
$h = 0$
$j = 3$

It is structure / truss (dof = 0) / frame used to transfer a load

$f = 4$
$h = 0$

d.o.f = 1 ⇒ It is kinematic mechanism

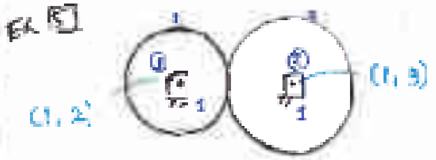
NOTE

- It is a constraint mechanism
- d.o.f = 1 ⇒ only one eqⁿ required betⁿ input & output
- d.o.f = 1 ⇒ only one link must controlled as input in order to have constrained mechanism.

NOTE

- If '1' is subtracted from the d.o.f freedom from any chain & any answer comes '1' then only the chain could be called as "kinematic chain"

Ex. 5



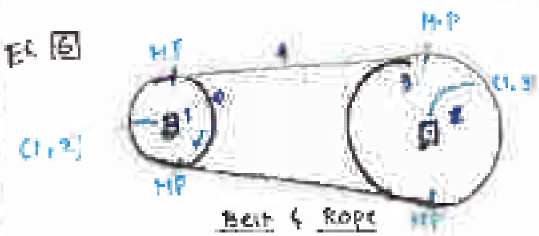
$n = 3$
$j = 2$
$h = 1$

$3(3-1) - 2(2) - 1$
$6 - 4 - 1$

$f = 3(n-1) - 2j - h$
$= 3(3-1) - 2(2) - 1$

$f = 1$ ← constraint seqⁿ & eqⁿ

Ex. 6

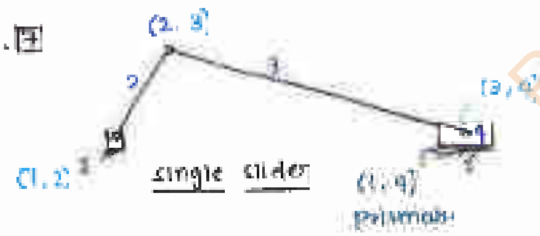


$n = 4$
$j = 2$
$h = 4$

$f = 3(4-1) - 2(2) - 4$

$f = 1$

Ex. 7

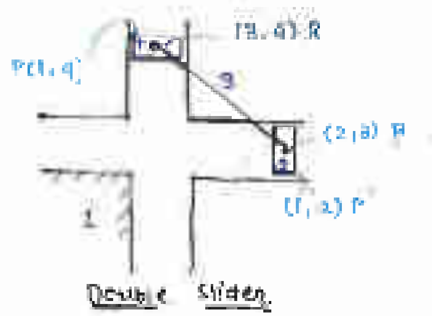


$n = 4$
$j = 3R + 1P = 4$
$h = 0$

$f = 3(4-1) - 2(4) - 0$

$f = 1$

Ex. 8

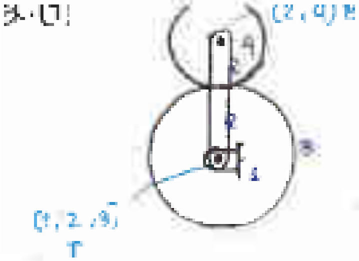


$n = 4$
$j = 2P + 2R = 4$
$h = 0$

$f = 3(4-1) - 2(4) - 0$

$f = 1$

Ex-17



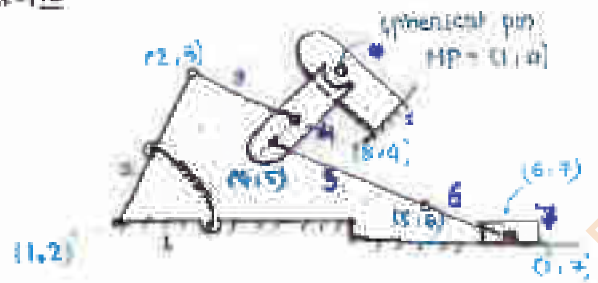
$j = LP + HT$
$= [illegible]$
$h = 1$
$dof = 3(4-1) - 2(3) - 1$
$\boxed{dof = 2}$

⇒ It is an unconstrained mechanism
d.o.f = 2 ⇒ Therefore two eqⁿ are required between input & output that is two links are working as output link

	i/p	o/p
unconstrained d.o.f = 2	3	2, 4
constrained d.o.f = 1	3	either 2 or 4

⇒ In an epicyclic gear train, arm will always be connected either with some input or some output

Ex-18



$n = 8$

$n = 7$
$j = 7$
$h = 1$
$d.o.f = 3(7-1) - 2(7) - 1$
$\boxed{dof = 3}$
3 eqⁿ required

⇒ Kinematic Diagrams of various link

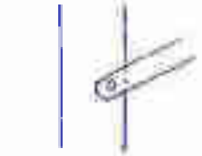
Actual	Kinematic

(a)

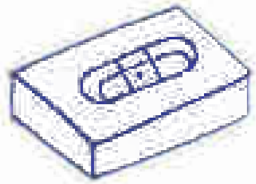
(b)

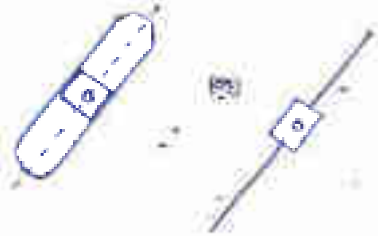
(c)

c)

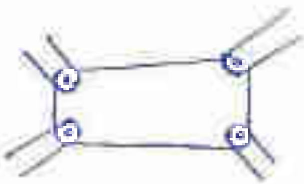


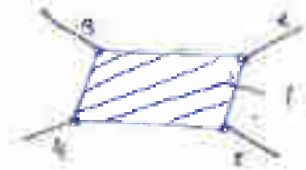
d)





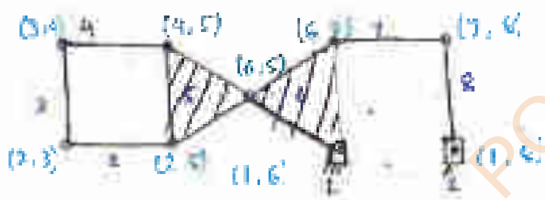
e)





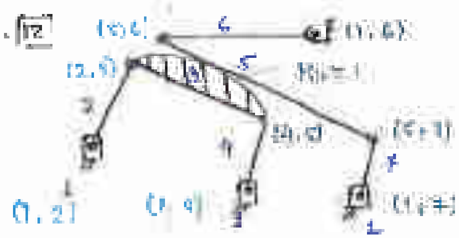
f)

Ex. 11



n = 8
j = 9
3(8 − 1) − 2(9)
21 − 18
d.o.f = 3

Ex. 12

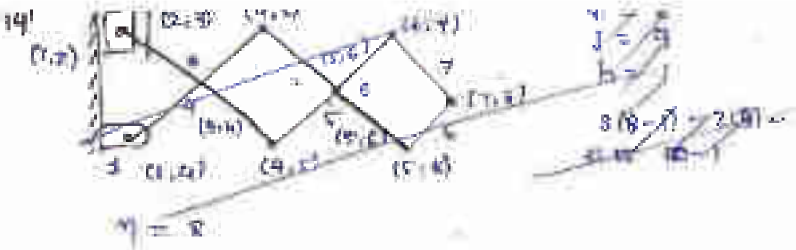


n = 7
j = 8
hp = 1
3(7 − 1) − 2(8) − 1
18 − 16 − 1
d.o.f = 1

Ex. 13

n = 4
j = 3
hp = 1
3(4 − 1) − 2(3)
9 − 6 − 1
f = 2

14'



n = 8

14

15

$n = 8$

$j = 2B + 3T + 1Q$

$= 2B + 6B + 3B$

$= 11$

$F = 21 - 22 = \boxed{-1}$

⇒ Exception to the Kutzbach equation

$$\boxed{D.o.f = 3(n-1) - 2j - h_p}$$

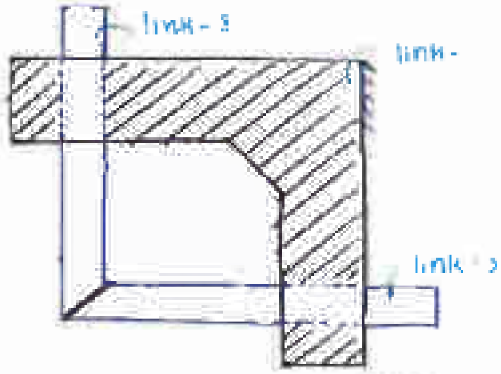
- Kutzbach equation is only valid for planer mechanism that is in which different point on different link move in parallel planes & consist of mainly Revolute pair and prismatic pair.
- There are some exceptions when Kutzbach equation get violated, in this cases we should use modified Kutzbach equation.

★ Modified Kutzbach Equation:

$$\boxed{D.o.f = 3[n - n_r - 1] - 2[j - j_r] - h - F_r}$$

Where: n = total no. of links
n_r = total no. of redundant link
j = total no. of binary pair joint
j_r = total no. of redundant joint
h = no. of h.p
F_r = redundant dof

In all the mechanism it consist of 3 links closed loop having lower pairs only



prismatic pair - 3

$n = 3$

$j = 3$

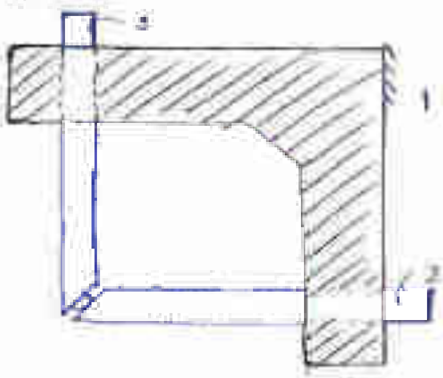
$h = 0$

$f = 3(n-1) - 2j - h$

$= 3(3-1) - 2(3) - 0$

$\boxed{f = 0}$

- It means it is a structure. None of the given linkage is able to transfer the relative motion from link - 2 to link - 3



$n = 3$

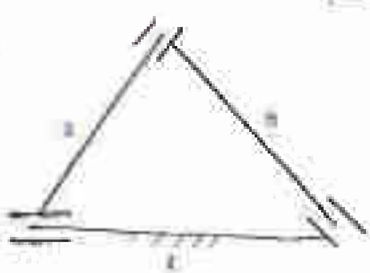
$j = 2$

$h_p = 1$

$f = 3(n-1) - 2j - h_p$

$= 3(3-1) - 2(2) - 1$

$\boxed{f = 1}$



$\boxed{f = 1}$

case-(ii): If a mechanism consist of redundant link:

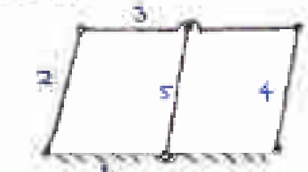
$l_1 = l_3$ and $l_1 \parallel l_3$

$l_2 = l_4$ and $l_2 \parallel l_4$

critical position

or uncertainty configuration

(i) As the link will become co-linear which leads to decrement in rigidity & chance of failure will be maximum corresponding to it.
In order to prevent the failure; corresponding to uncertainty configuration, we use redundant link in mechanism & it should be connected to parallel some link.



Double $\| ^{gm}$ Linkage

Simple Kutzback eqn

$n = 5$
$j = 6$
$h = 0$

$f = 3(5-1) - 2(6) - 0$

$\boxed{f = 0}$

Modified Kutzback

$D.O.F = 3[n - n_r - 1] - 2[j - j_r] - h - f_r$

$= 3[5 - 1 - 1] - 2[6 - 2] - 0 - 0$

$\boxed{f = 1}$

→

$\boxed{D.O.F = 1}$

NOTE:

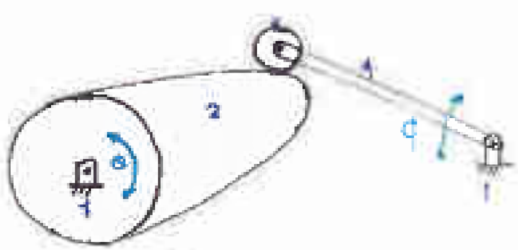
E / m linkage having $\boxed{f = 1}$

→

$\boxed{f = 0}$

because link '5' is not $\|$ to 2 & 4

case-(iii): The mechanism which consist of redundant pair



$n = 4$
$j = 4$
$h = 1$

$f = 9 - 8 - 1$

$\boxed{f = 1}$

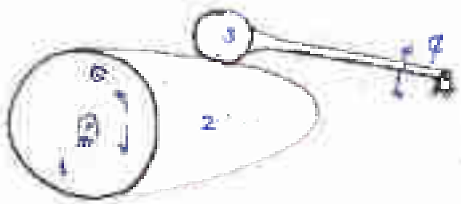
Oscillating cam & follower mechanism

Hence, we require only one equation between input & output therefore d.o.f for cam & follower is actually '1'.

θ = [illegible] of cam
ϕ = oscillation of follower

Explanation (a):

- If we welded the follower's joint 3



$f = 3(3-1) - 2(2) - 1$

$\boxed{f = 1}$

- D.o.f by modified kutzback eq^n

$f = 3[n - n_R - 1] - 2[j - j_R] - h - f_R$

$= 3[4 - 1 - 1] - 2[3 - 1] - 1$

$\boxed{f = 1}$

Explanation (b)

- Mechanism consist of redundant degree of freedom.
If pairs would be redundant like '3' but not joint redundant

$f = 3[n - n_R - 1] - 2[j - j_R] - h - F_R$

$= 3[4 - 0 - 1] - 2[3 - 0] - 1 - 1$

$\boxed{f = 1}$

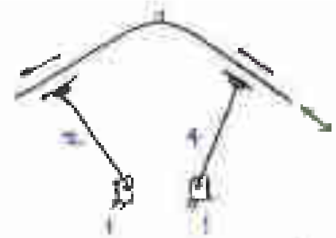
- The spinning motion of roller is redundant. Hence cam & follower mechanism consist of one redundant d.o.f
- cam & follower are constrained mechanism.

Case (iv): Mechanism consisting of surface motion pair (or) redundant d.o.f.
surface motion is d.o.f > 1

$d.o.f = 3[n - n_R - 1] - 2[j - j_R] - h - F_R$

$= 3[4 - 0 - 1] - 2[4 - 0] - 0 - 1$

$\boxed{f = 0}$



d.o.f =

Note:

(i) According to Grubler's criteria min. no. of links required to make a mini mechanism is '4'

(ii) A mechanism which consist of all the '5' prismatic pairs is possible (case-(i))

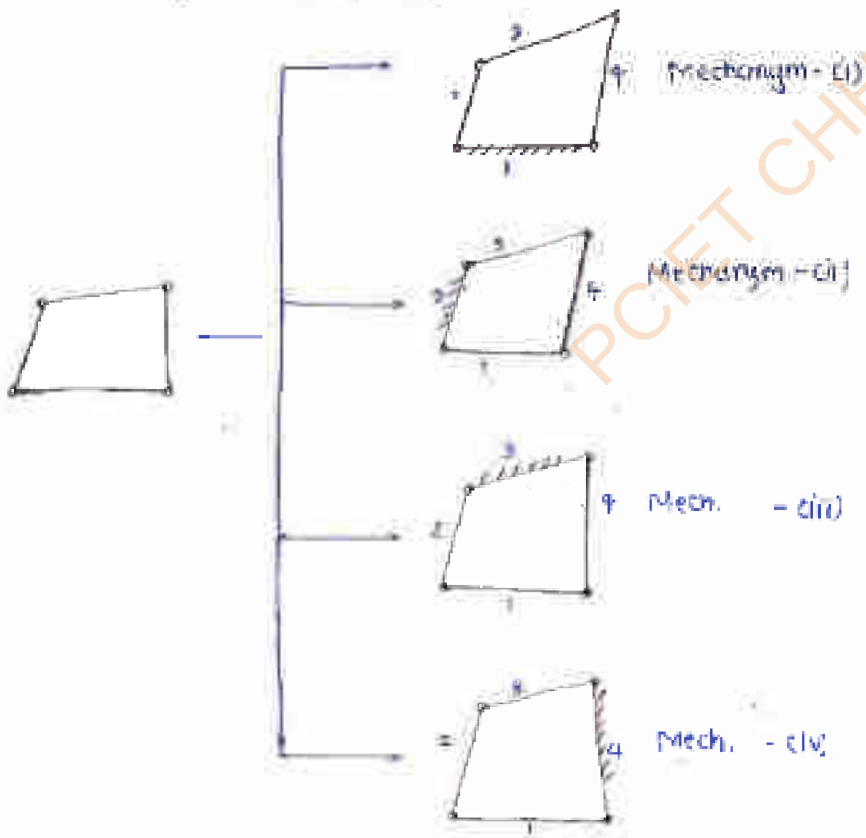
(iii) Minimum '3' links are required to make a mechanism consisting of atleast one higher pair.

Ex. Gear & pinion mechanism

⟹ Inversion of a Mechanism:

(purpose of inversion is to analyse easily the mechanism problems)

– The process of fixing different links of a mechanism is known as inversion of mechanism



- If there are 'n' no. of links then possible inversion will also be n.
- Inversion of mechanism does not change the relative motion between links. It is the characteristic of parent kinematic chain but inversion do affect the absolute motion various links.
- Inversion are used to make the problem simple in some c
  Ex. cam & follower mechanism
  sun & planet gear train
  → Higher pair can not be inverted.

⇒ **Range of Movement :-**

(or) **Grashof's Law :-**

- On the basis of type of movement the links are classified as followed.

i) Frame / fixed link
→ which does not move.

ii) Crank :
→ The link which is able to execute full circular motion or which can rotate completely.

iii) Rocker / lever
→ The link which can not rotate completely that is the one wh is oscillate.

iv) Coupler
→ The link which is opposite to frame of the link which connects input to the output.

- On the basis of relation between dimensions of various links 4-bar mechanisms are classified in 3 categories.

⇒ Class - I Linkage

$l_{min} + l_{max} < p + q$ Grashof's Linkage

⇒ Class - II Linkage

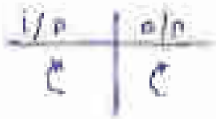
$l_{min} + l_{max} > p + q$ Non Grashof's Linkage (NGL)

⇒ Class - III Linkage

$l_{min} + l_{max} = p + q$ Transition Linkage or special Grashof's Linkage

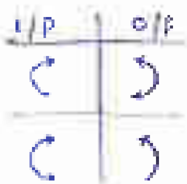
- The position of shortest link is always decide the type of inversion to Grashof linkage.

→ Inversion-(a): If shortest (l_{min}) link is fixed
- The input & output both will be able to execute full circular motion.



Crank - crank
(or)
Double crank
(or)
Drag link mechanism

→ Inversion-(b): If shortest (l_{min}) is adjacent to fixed



Crank Rocker
(or)
Rocker crank

→ Inversion-(c): If shortest (l_{min}) link is opposite to the fixed
(or) l_{min} is coupler

Rocker Rocker
(or)
Double Rocker
(or)
Lever mechanism

⇒ Inversion of class - II linkage:
- All the possible inversion of non grashof's linkage double rocker only.

⇒ Inversion of class - III linkage
- inversion of transition linkage will follow the inversion of Grashof linkage.

$l_{min} + l_{max} = p + q$

All the links are of equal length

$l_{min} = l_{max} = p = q$

e.g.

Rhombus linkage

Two links are equal length

$l_{min} = p$

$l_{max} = q$

Equal length links are parallel to each other

Equal length links are adjacent to each other

Deltoid linkage
kite linkage

Galloway linkage

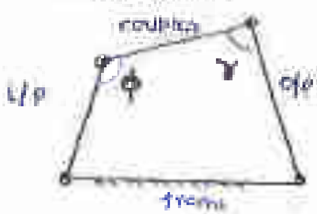
VR = $\frac{1}{2}$

If (l_{min} = p) fixed
Double crank

If shorter adjacent to ...
Crank - Rocker
or Rocker - ...

⟹ Transmission angle [γ]

- It is the parameter to indicate the effectiveness of lower pair mechanism
- It is acute angle (γ < 90°) formed between coupler & output link

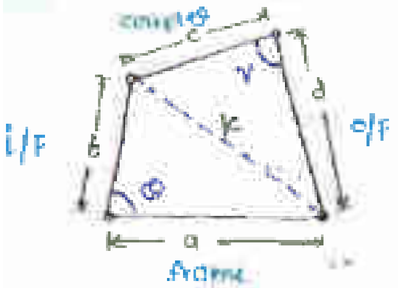


⟹ Pressure angle (ϕ)

→ It is the angle between input & coupler.

→ Pressure angle is mainly used as a parameter of effectiveness in higher pair mechanism

here k follows



$$k^2 = a^2 + b^2 - 2ab\cos\theta$$

$$k^2 = c^2 + d^2 - 2cd\cos\gamma$$

$$a^2 + b^2 - 2ab\cos\theta = c^2 + d^2 - 2cd\cos\gamma$$

$$\boxed{2cd\cos\gamma = c^2 + d^2 - a^2 - b^2 + 2ab\cos\theta}$$

$$\gamma = f(\underbrace{a, b, c, d}_{const}, \theta)$$

→ $\gamma = g(\theta)$

for max^n or min^n

$$\frac{d\gamma}{d\theta} = 0$$

$$\Rightarrow 2cd(-\sin\gamma)\frac{d\gamma}{d\theta} = 0 + 2ab(-\sin\theta)$$

$$\boxed{\frac{d\gamma}{d\theta} = \frac{ab}{cd}\cdot\frac{\sin\theta}{\sin\gamma}}$$

$$\rightarrow \frac{ab}{cd}\,\frac{\sin\theta}{\sin\gamma} = 0$$

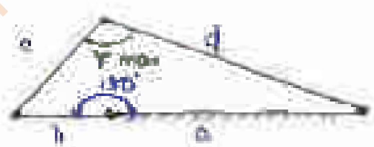
$$\Rightarrow \frac{\sin\theta}{\sin\gamma} = 0$$

$$\boxed{\sin\theta = 0} \Rightarrow \boxed{\begin{array}{l}\theta = 0^\circ \\ \theta = 180^\circ\end{array}}$$

for $\theta = 0^\circ$:

$$(a-b)^2 = c^2 + d^2 - 2cd\cos\gamma_{min}$$

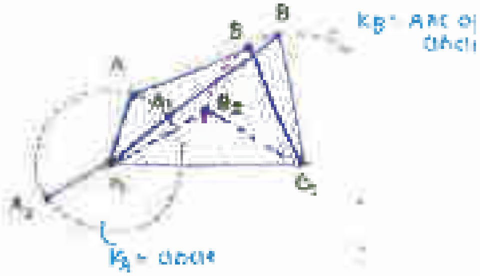
for $\theta = 180^\circ$



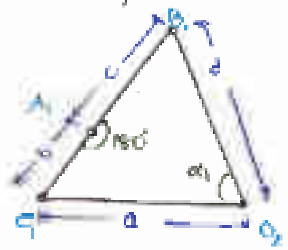
$$(a+b)^2 = c^2 + d^2 - 2cd\cos\gamma_{max}$$

Note :

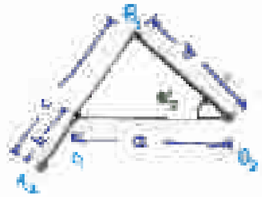
(i) $\theta = 0^\circ$ & 180° leads to min. transmission angle and it is possible either in double crank mechanism or crank rocker mechanism



for $\phi = 180^\circ$

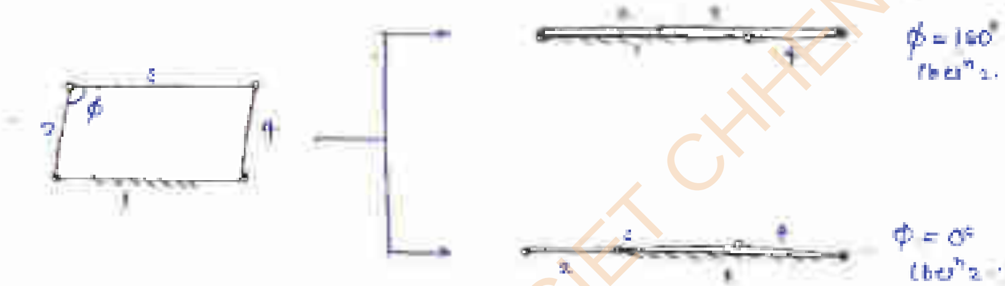


for $\phi = 0^\circ$



NOTE:

In parallelogram linkage



Mechanical Advantages [MA]

- It is analogous to efficiency of the engine
- Mechanical advantages are defined as the ratio of torque at output link to the input link torque

$$MA = \frac{\text{Torque @ O/P}}{\text{Torque @ I/P}}$$

$$MA = \frac{T_4}{T_2}$$

- If there is no power loss;

power @ I/P = power @ O/P

$$T_{in}\,\omega_{in} = T_{out}\,\omega_{out}$$

$$\boxed{\frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} = MA}$$

MA ... ω_{in}

$$\boxed{VR = \frac{\omega_{out}}{\omega_{in}} <} \Rightarrow \boxed{MA = \frac{1}{VR}}$$

→ $MA = f\left[\frac{\sin \gamma}{\sin \phi}\right]$

→ Corresponding to toggle position $MA = \infty$

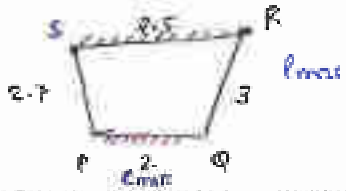
Hooke joints (OBS)

→ It is universal joint
→ It is spatial mechanism (3D) ie
→ Non parallel - non planar

CRQ-41

$\boxed{VR = 1}$

[5]



It is class - ② problem

$l_{min} + l_{max} = 2 + 3.5 = 5.5$

$p + q = 2.7 + 3 = 5.7$

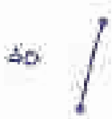
$l_{min} + l_{max} < p + q$

Shortest link

[RS]

[6]

$l_f = 20$ ←
$l_{in} = 40$
$l_o = 50$
$l_{ow} = 60$



$l_{min} + l_{max}$ □ $p + q$

$20 + 60 < 50 + 40$

class - I problem.

Shortest link whi decide ⇒ Full circle

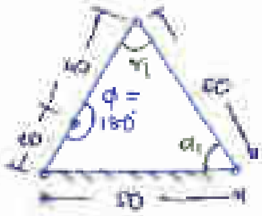
18)

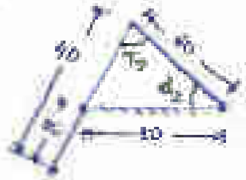
$l_c = 20$ ✓
$l_{max} = 40$
$l_{max} = 60$ ✓

$l_{min} + l_{max}$ ☐ $P + Q$
$20 + 60$ ☐ $50 + 40$
$80 < 90$

class – (1)

→ Fixed link → extreme position
$\phi = 0°$ or $\phi = 180°$





$\phi = 180°$

$(20+40)^2 = (50)^2 + (60)^2 - 2(50)(60)\cos\alpha_1$

$\boxed{\alpha_1 = 65.3°}$

$(40-20)^2 = (50)^2 + (60)^2 - 2(50)(60)\cos\alpha_2$

$\boxed{\alpha_2 = 18.19°}$

min transmission angle
$\gamma_1 = ?$
$(50)^2 = (20+40)^2 + (60)^2 - 2(60)(60)\cos\gamma_1$

$\boxed{\gamma_1 = 49.248°}$

for $\gamma_2 = ?$
~~$(40-20)^2 = 50^2 + 60^2$~~
$50^2 = (40-20)^2 + 60^2 - 2(20)(60)\cos\gamma_2$

$\boxed{\gamma_2 = 57.8°}$

for Maximum angle through it oscillating.

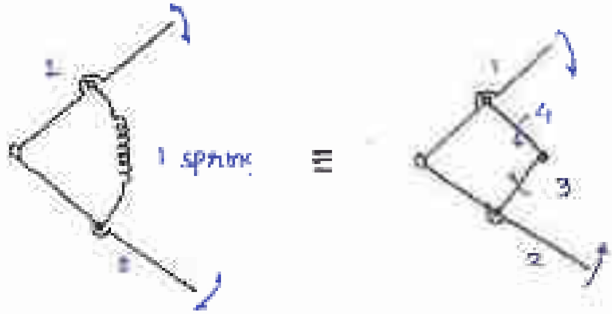
$\alpha_1 - \alpha_2 = 47.18°$

total dist travel by rocker = $2(\alpha_1 - \alpha_2)$

19)

$(10\sqrt{29})^2 = (40)^2 + (60)^2 - 2(40)(60)\cos\gamma$

$\boxed{\gamma = 61.39}$



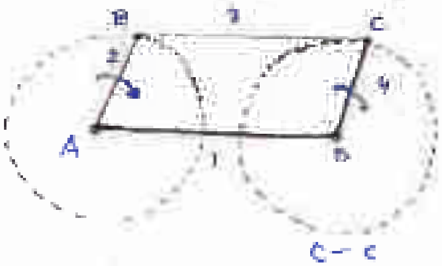
| 1 spring = 2 Binary link |

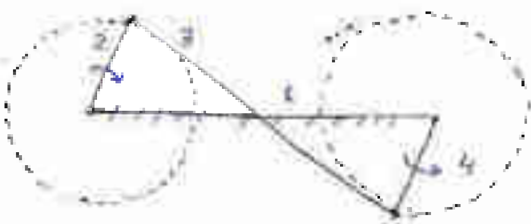
→ Higher pairs

n = 3
j = 2
h = 1

Double - crank
or
Drag link mechanism

→ convert in parallelogram

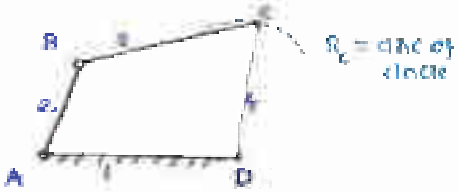


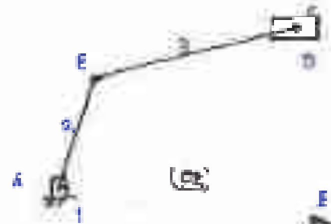


n = 4
j = 4
h = 0

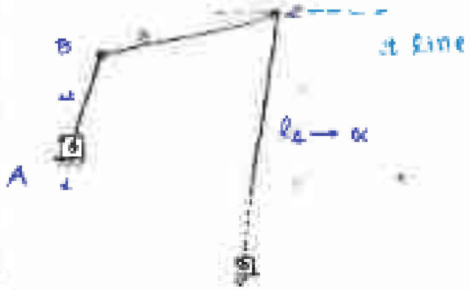
1 pair = 1 link + 2 Binary joint

| 1 higher pair = 2 lower pair |



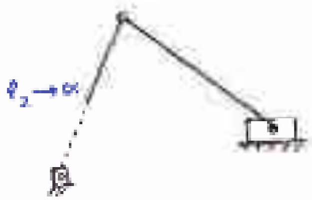


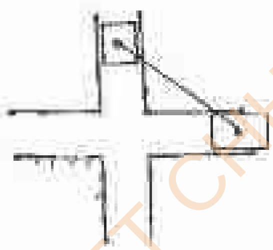
l_4 will oscillate & for making it straight line



→ since straight line is a part of circle whose center lies at infinite.

→ Double slider mechanism :-





[*] Inversion single slider mechanism :

Kinematic chain

$l_{min} + l_{max}$ [<] $p + q$

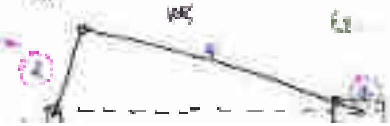
$l_2 + \infty$ [<] $l_3 + \infty$

so it belongs class I mechanism

Inversion-(I) Link 1 is fixed

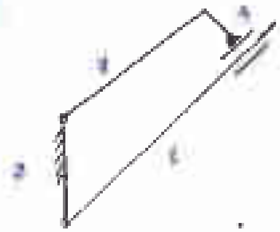
→

(1) offset slider crank GRP



(2) IC engine, Reciprocating compressor

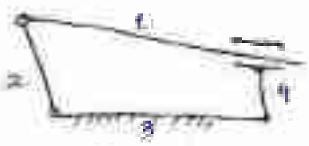
→ If input link is [illegible] it becomes compressor [illegible]



i/p	o/p
ζ	ζ

Ex: with Wonth Quick Return mechs

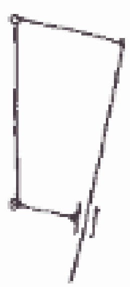
Inversion-(iii) If link-3 is fixed i.e. CR is fixed



crank - rocker

→

Ex: oscillating cylinder engine mechanism



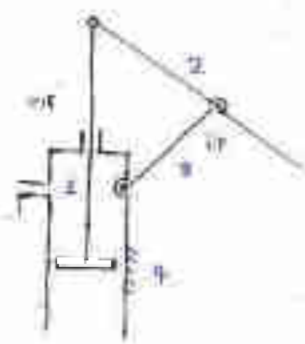
(or

Ex: crank & slotted bar QRM

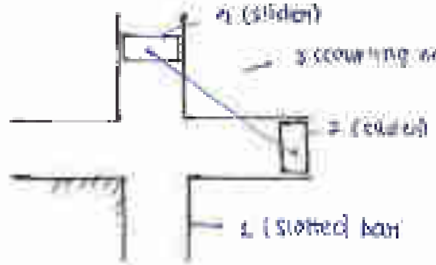
Inversion-(iv) If slider fixed i.e. link 4 is fixed

Rocker - Rocker

≗



Ex: Hand pump



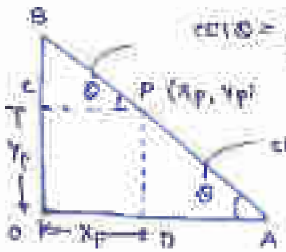
$l_{min} + l_{max} < p + q$

$l_{min} + \infty < \infty + \infty$

Inversion - (I) Link-1 i.e. slotted bar is fixed

→ Rocker - Rocker / Lever mechanism

Here rocker is opposite link-3 is fixed become Rocker - Rocker



$\cos\theta = \frac{PC}{BP} \Rightarrow \cos\theta = \frac{x_p}{BP}$

$\sin\theta = \frac{PD}{PA} \Rightarrow \sin\theta = \frac{y_p}{AP}$

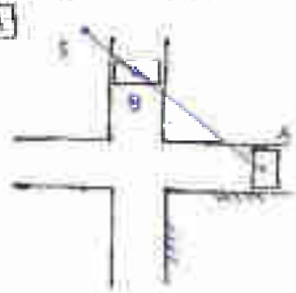
$$\boxed{\left(\frac{x_p}{BP}\right)^2 + \left(\frac{y_p}{AP}\right)^2 = 1}$$

→ Locus of P = ellipse

→ Elliptical trammel

→ special case

[A]



K_1 = Ellipse

→ K_2 = ellipse

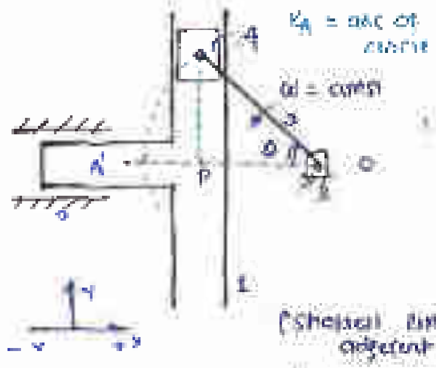
[B] If P is midpoint of AB :

$AP = BP \Rightarrow \frac{x_p^2}{(AP)^2} + \frac{y_p^2}{(AP)^2} = 1 \Rightarrow \boxed{x_p^2 + y_p^2 = (AP)^2}$

→ circle

→ K_P = straight line

→ Inversion (iii): If slider is fixed. (Scotch-yoke mechanism)



displacement of slotted bar

$x = PA'$

$= OA' - OP$

$= OA - OA\cos\theta$

$$x = OA(1-\cos\theta)$$

(Slotted link adjacent line)

→ crank-rocker (link-4 is fixed, link-1 is adjacent to fixed is crank rocker)

→ Velocity (v):

$$v = \frac{dx}{dt} = \frac{d}{dt}\{OA(1-\cos\theta)\}$$

$$= OA\left[0-(-\sin\theta)\frac{d\theta}{dt}\right]$$

$$v = OA \cdot \omega \cdot \sin\theta$$

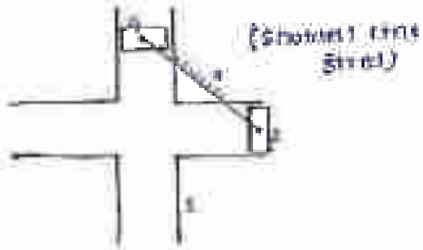
→ Accⁿ (a):

$$a = \frac{dv}{dt} = OA\ \omega \cos\theta \frac{d\theta}{dt}$$

$$a = OA \cdot \omega^2 \cdot \cos\theta$$

Scotch yoke mechanism

SHM ⟺ oscillation



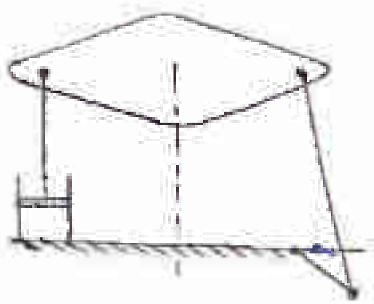
Oldham coupling

→ Double crank

– Oldham's coupling is used for connect two shaft which are having parallel misalignment

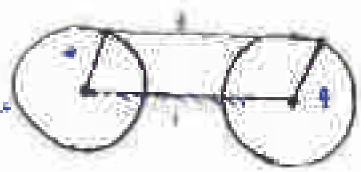
⟹ Inversion of simple four bar mechanism

Inversion - (a)



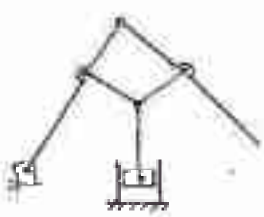
Ex Beam engine (or) watt engine

Inversion - (b)



coupling rod of locomotive

Inversion - (c)



watt's indicator diagram

- A QRM is mechanism in which return stroke consumes less time than cutting stroke since return stroke is idle & it should occur as fast as possible whereas cutting is main working stroke & maximum energy consumption occurs during this stroke.
- For all QRM we define a Quick Return Ratio as follows

$$Q.R.R. = \frac{\text{time required in cutting stroke}}{\text{time required in return stroke}}$$

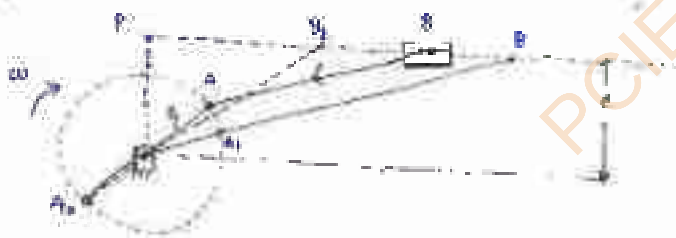
→ If angular speed of driver is constant
i.e. $\theta = \omega t \Rightarrow \boxed{\theta \propto t}$

$$QRR = \frac{\text{Angular dis}^n \text{ travelled in cutting stroke } (\alpha)}{\text{Angular dist}^n \text{ travelled in return stroke } (\beta)}$$

$$\boxed{QRR = \frac{\alpha}{\beta} > 1}$$

Note:
If QRR given less than < 1
then $\boxed{QRR = \frac{\beta}{\alpha} < 1}$

Offset slider crank quick return mechanism:



Stroke length $= B_1B_2$
$= PB_1 - PB_2$

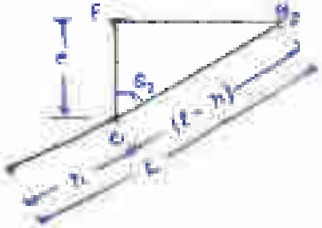
In ΔOB_1P

$\sin\theta_1 = \frac{B_1P}{OB_1} \Rightarrow B_1P = OB_1 \sin\theta_1$

$\boxed{B_1P = (l+r)\sin\theta_1}$

$\cos\theta_1 = \frac{OP}{OB_1}$

$\boxed{\cos\theta_1 = \frac{e}{l+r}}$



$\sin\theta_2 = \frac{PB_2}{OB_2} = \frac{PB_2}{l-r}$

$PB_2 = (l-r)\sin\theta_2$

$\cos\theta_2 = \frac{e}{l-r}$

$\Rightarrow$ Stroke length $= (l+r)\sin\theta_1 - (l-r)\sin\theta_2$

→ QRR = $\frac{\text{angle turned in cutting stroke}}{\text{angle turned in return stroke}}$

$QRR = \frac{180°+\phi}{180°-\phi}$

where; $\phi = \theta_1 - \theta_2$

PRO-10) $r = 20$ cm
$l = 40$ cm
$e = 10$ cm

$\cos\theta_1 = \frac{e}{l+r} = \frac{10}{20+40}$

$\theta_1 = 80.40°$

$PB_1 = (l+r)\sin\theta_1$
$= (20+40)\sin 80.40$
$PB_1 = 59.16$ cm

$\cos\theta_2 = \frac{e}{l-r} = \frac{10}{40-20}$

$\theta_2 = 60°$

$PB_2 = (l-r)\sin\theta_2$
$= (40-20)\sin 60$
$= 17.32$

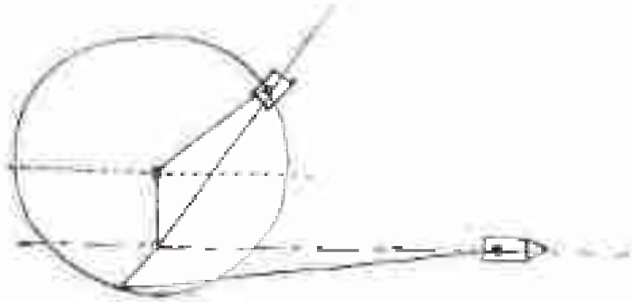
Stroke $= PB_1 - PB_2$

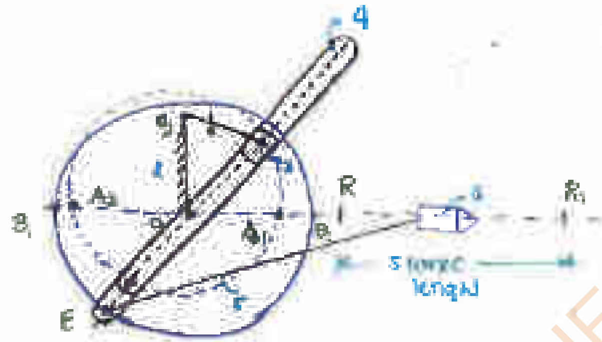
Stroke $= 41.84$

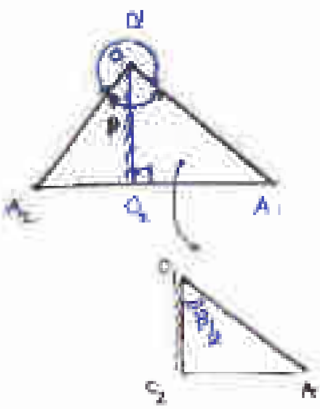
$\phi = \theta_1 - \theta_2 = 80.40 - 60$

— $QRR = \frac{180°+\phi}{180°-\phi} = \frac{180°+20.405}{180°-20.405}$
$= \frac{200.405}{159.595}$

$QRR = 1.25$







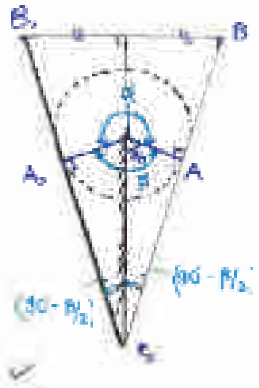
$$QRR = \frac{\alpha}{\beta} \ (>1)$$

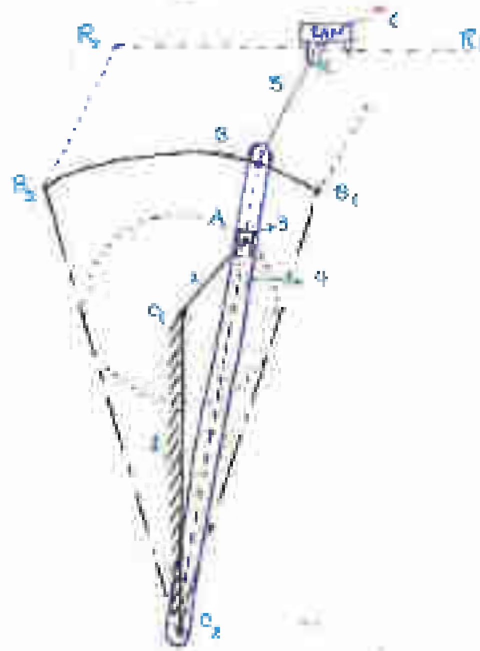
$$\alpha + \beta = 360$$

$$\cos\frac{\beta}{2} = \frac{O_2O_1}{O_1A}$$

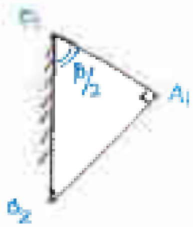
$$\cos\frac{\beta}{2} = \frac{\text{fixed link length}}{\text{slotting crank length}}$$

$$\text{stroke length} = R_1R_2 = B_1B_2$$





→ Extreme position



$\cos \beta/2 = \frac{O_1A_1}{O_2A_1} = \frac{O_1A_1}{O_1O_2}$

$$\cos \frac{\beta}{2} = \frac{\text{crank length}}{\text{fixed link length}}$$

$$QRR = \frac{\alpha}{\beta} > 1$$

; $\alpha + \beta = 360$

→ Stroke length :

stroke length $= R_1R_2$

$= B_1B_2$

$= B_1P + B_2P$

$= 2B_1P$

$= 2O_2B_1 \sin(90 - \beta/2)$

$= \sout{2 \cdot O_2B_1 \sin \beta/2}$

$= 2 \cdot O_2B_1 \cdot \cos \beta/2$

$$\text{Stroke length} = 2 \times \text{length of slotted bar} \times \frac{\text{crank length}}{\text{fixed link length}}$$

→ Use → shaper M/C

$l = 40$ cm

$\cos \beta/2 = \frac{\text{crank length}}{\text{fixed length}} = \frac{20}{40}$

$\cos \beta/2 = 20/40 = 1/2$

$\frac{\beta}{2} = 60 \Rightarrow$ $\boxed{\beta = 120^\circ}$ $\Rightarrow$ $\boxed{\alpha = 240}$

$QRR = \frac{\alpha}{\beta} = \frac{240^\circ}{120^\circ} \Rightarrow$ $\boxed{QRR = 2}$

[46] $QRR = \frac{1}{2} = \beta/\alpha \Rightarrow \alpha + \beta = 360$

$\cos \beta/2 = 1/2 =$

NOTE :

→ If QRR = 2 : 1 (or) 1 : 2 crank length always be half of the fixed link length

(i) Analytical Approach
- Vector Algebra
- Complex No

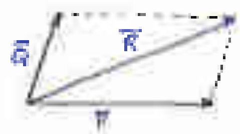
(ii) Graphical Approach
- Instantaneous centres of rotation (velocity) [I-centres]
- Velo' & Acc' diagrams

★ Vectors

$\vec{a} = |\vec{a}| \cdot \hat{a}$

$|\vec{a}|$ → magnitude; $\hat{a}$ → unit vector (direction)

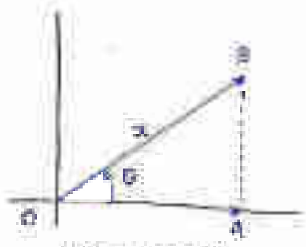
→ $\vec{P} + \vec{Q} = \vec{R}$



$|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

→ $\vec{AB} \neq \vec{BA}$

but $\vec{AB} = -\vec{BA}$



$OA = x\cos\theta$

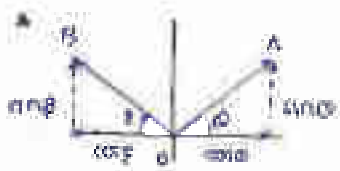
$OB = x\sin\theta$

$\vec{OB} = \vec{OA} + \vec{AB}$

$|\vec{OB}| \cdot \hat{OB} = |\vec{OA}| \cdot \hat{OA} + |\vec{AB}| \cdot \hat{AB}$

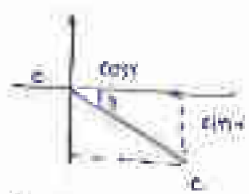
$x \cdot \hat{OB} = (x\cos\theta)\hat{i} + (x\sin\theta)\hat{j}$

$$\boxed{\hat{OB} = \hat{i}\cos\theta + \hat{j}\sin\theta}$$

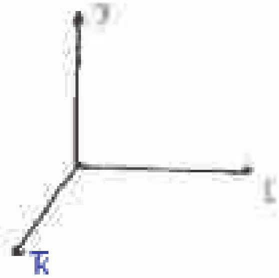


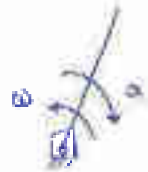
$\hat{OA} = \hat{i}\cos\alpha + \hat{j}\sin\alpha$

$\hat{OB} = -\hat{i}\cos\beta + \hat{j}\sin\beta$



$\hat{OC} = \hat{i}\cos\gamma - \hat{j}\sin\gamma$





$\vec{\omega} = \omega \hat{k}$ (following right hand rule)

$\vec{\alpha} = \alpha(-\hat{k})$

Tangential direction $\hat{k}$

$\hat{i} \times \hat{j} = \hat{k}$

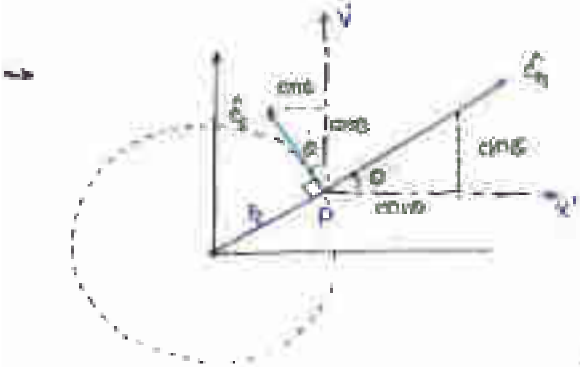
$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{i} = -\hat{k}$

(Reverse cyclic product give (-ve) results)



$\hat{e}_r = \hat{i}\cos\theta + \hat{j}\sin\theta$

$\hat{e}_t = \hat{j}\cos\theta - \hat{i}\sin\theta$

Displacement eq^n

$\vec{OP} = |\vec{OP}| \hat{OP}$

$\vec{OP} = r \cdot \hat{e}_r$

$\vec{OP} = r(\hat{i}\cos\theta + \hat{j}\sin\theta)$

⇒ Velocity Equation

$\vec{v} = \frac{d\vec{OP}}{dt} = \frac{d}{dt}[r(\hat{i}\cos\theta + \hat{j}\sin\theta)]$

$= r\frac{d}{dt}[\hat{i}\cos\theta + \hat{j}\sin\theta]$

$= r\left[\hat{i}(-\sin\theta)\frac{d\theta}{dt} + \hat{j}(\cos\theta)\frac{d\theta}{dt}\right]$

$\vec{v} = r\omega[-\hat{i}\sin\theta + \hat{j}\cos\theta]$

$\vec{v} = (r\omega)\hat{e}_t$

mathematically:

$\vec{v} = \vec{\omega} \times \vec{r}$

$V_A = (OA\cdot\omega)\,\hat e_t$

Note

① Relative velocity have only component

② The component will always be perpendicular to the line

★ Acceleration Eq^n

$$\vec a = \frac{d\vec v}{dt} = \frac{d}{dt}\left[r\omega(-\hat i\sin\theta + \hat j\cos\theta)\right]$$

$$= r\omega\frac{d}{dt}\left[-\hat i\sin\theta + \hat j\cos\theta\right]$$

$$= r\omega\left[-\hat i\cos\theta\frac{d\theta}{dt} + \hat j\left(-\sin\theta\frac{d\theta}{dt}\right)\right]$$

$$+ (-\hat i\sin\theta + \hat j\cos\theta)\cdot\frac{d}{dt}(r\omega)$$

$$= r\omega^2(-\hat i\cos\theta - \hat j\sin\theta) + (-\hat i\sin\theta + \hat j\cos\theta)\left[r\frac{d\omega}{dt}\right]$$

$$\boxed{\vec a = r\omega^2(-\hat e_r) + r\alpha(\hat e_t)}$$

→ Acceleration has got two components

① normal

② tangential

$\vec a_{normal} = r\omega^2(-\hat e_r)$ (Radial)

$\vec a_n = \vec\omega\times(\vec\omega\times\vec r)$

→ Direction of normal acc^n will always be towards the centre of rotation.

$\vec a_{tangential} = r\alpha(\hat e_t)$

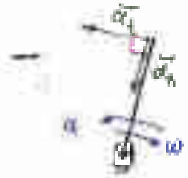
$\vec a_t = \vec\alpha\times\vec r$

tangential acceleration is always take along the tangent.

→ Normal acc^n is always perpendicular to tangential.

$a = a_{radial} + a_{tangential}$

$\vec{a} = OA\,\omega^2(-\hat{e}_r) + OA\,\alpha\,(\hat{e}_t)$



→ $\vec{a}_{rad}$ & $\vec{a}_{t}$ is always perpendicular to

$$|\vec{a}_R| = \sqrt{[|\vec{a}_n|]^2 + [|\vec{a}_t|]^2}$$

$$a_R = \sqrt{a_n^2 + a_t^2}$$

→ Rolling Motion

Rolling = Translation + Rotation

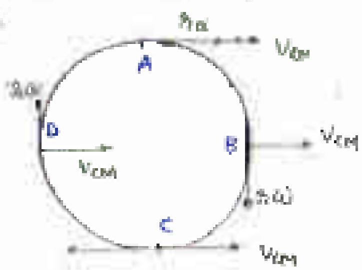
for velocity :

mass distribution is not important

Distribution of mass important

$$V_A = V_B = V_C = V_D = V_{CM}$$

Resultant :



@ point C

$V_{CM} = r\omega$

$$V_{point\ C} = 0$$

→ pure rolling

→ Rolling without slipping

$V_{CM} \neq r\omega$

Skidding: $V_{CM} > r\omega$

→ Striking on ice road

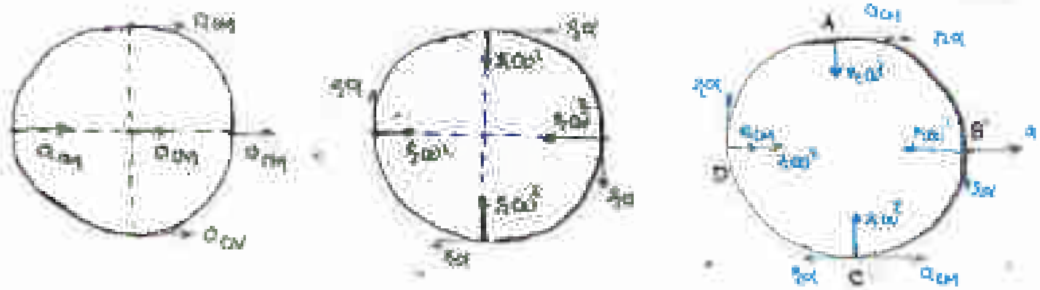
forward slipping

Slipping: $V_{CM} < r\omega$

→ Bike or wheel stuck in mud

Back ward slipping

acceleration



at point C

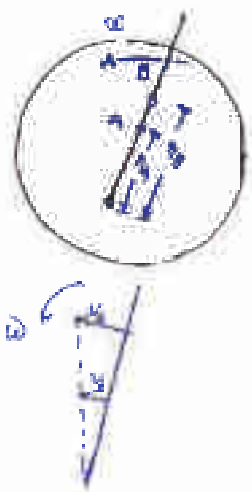
$a_{CM} = r\alpha$

$a_{point\ C} = r\omega^2$ (accn)

→ pure rolling

Rolling without slipping

Q - HS
& HG



$|\vec{V}_{BA}| = \vec{V}_B - \vec{V}_A$

$= |\vec{V}_B| - |\vec{V}_A|$

$= r_B\omega - r_A\omega$

$= (r_B - r_A)\omega$ (dir^n of comp. of dir^n of ...

$|\vec{a}_{BA}| = r_B\omega^2 - r_A\omega^2$

$= (r_B - r_A)\omega^2 \ (-\hat{e}_r)$ (towards center o...

→ property = f(space, time)

unsteady → prop. = f(time)

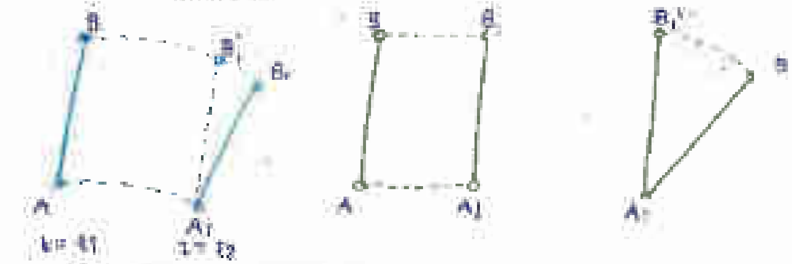
steady → prop ≠ f(time)

prop = f(space) → Non uniform

prop ≠ f(space) → Uniform

Instantaneous Centre of Rotation or I-Centre

General motion = Translation + Rotation



- In pure rotation there will be some finite centre of rotation.
- Since straight line is part of circle; whose centre lies at infinite. This definition enables us to define translation as an example of rotation whose rotation centre is at infinite.
- Hence we can conclude that every general motion is a kind of rotation and centre of rotation will be known as Instantaneously centre or I-centre.

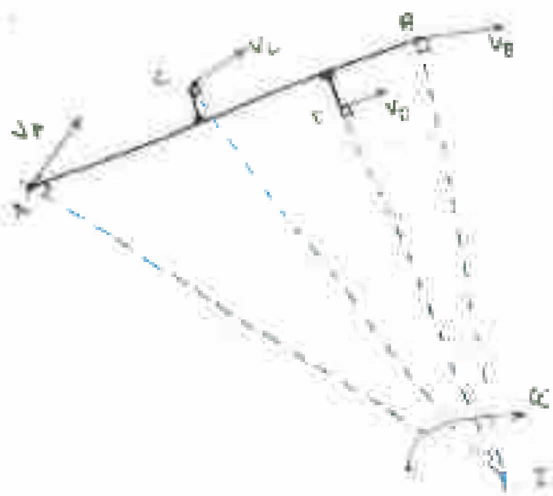
$$\vec{V}_A = \vec{V}_O + \vec{V}_{A/O} \qquad \vec{V}_{A/O} = \vec{V}_A - \vec{V}_O$$

$$= 0 + OA\cdot\omega\,\hat{e}_t$$

$$\boxed{\vec{V}_A = (OA\cdot\omega)\,\hat{e}_t}$$

$|\vec{V}_A| = CA\,\omega$ → ang. speed of link on which A exists

C: centre of rotation; A: the point whose velocity is to be calculated



$$V_A = IA\,\omega_{AB}$$

$$V_B = IB\,\omega_{AB}$$

$$\boxed{\frac{V_A}{IA} = \frac{V_B}{IB} = \frac{V_C}{IC} = \frac{V_D}{ID} + \dots = \omega_{AB} = const}$$

(a) If two links are connected with revolute pair the R-pair will itself become I-center.

(b) If two links are connected with prismatic pair the I-center will lie at infinite.

(c) If a link is on pure rolling motion over another link then point of contact will become an I-center.

→ If we consider Rotation + translation center of mass Rotation center of mass of disc & mass moment inertia should be considered about center of mass.

$K.E = Rot\ K.E$

$= \frac{1}{2} I_P \omega^2$

$I_P = I_c + mr^2$

$= \frac{mr^2}{2} + mr^2 = \frac{3}{2} mr^2$ (Disc)

$= \frac{1}{2} \times \frac{3}{2} mr^2 \omega^2$

$K.E = \frac{3}{4} mr^2 \omega^2$ (Translation only)

$K.E = Trans.\ K.E + Rot\ K.E$

$= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

$= \frac{1}{2} m (r\omega)^2 + \frac{1}{2} \left(\frac{mr^2}{2}\right) \omega^2$

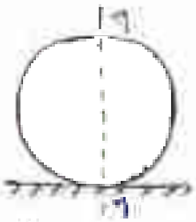
$\left\{ I = \frac{mr^2}{2} \right.$ (Disc)

$= mr^2\omega^2 \left[\frac{1}{2} + \frac{1}{4}\right]$

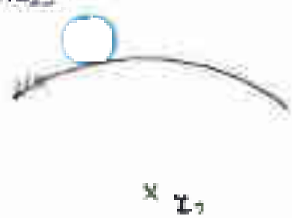
$K.E = \frac{3}{4} mr^2 \omega^2$ (Translation + Rotational drive)

② I-centre will lie somewhere along the common normal at the point of contact.

Ex com & follower



③ If disc is moving on a curved surface then centre of curvature becomes an I-centre.



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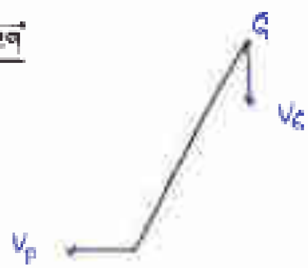
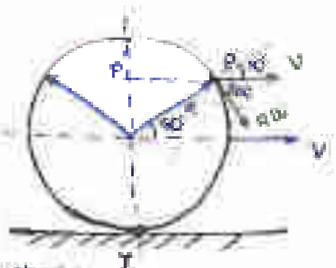


$\vec{V_Q} = \vec{V_P} + \vec{V}_{Q/P}$ $\qquad V_A = OA \cdot \omega$

$\therefore \vec{V}_{Q/P} = (PQ \cdot \omega_{PQ})\hat{e}_t$

(V_{QP} has one component perpendicular to PQ)

30



Disc rolls without slip

$V = R\omega$

$V_{rel} = \sqrt{V^2 + (R\omega)^2 + 2(V)(R\omega)\cos\theta}$

$= \sqrt{V^2 + V^2 \cdot 2V^2 \cdot \frac{1}{2}}$

$V_{rel} = \sqrt{3}\,V$

I-method

$\frac{V_P}{IP} = \frac{V}{IR}$

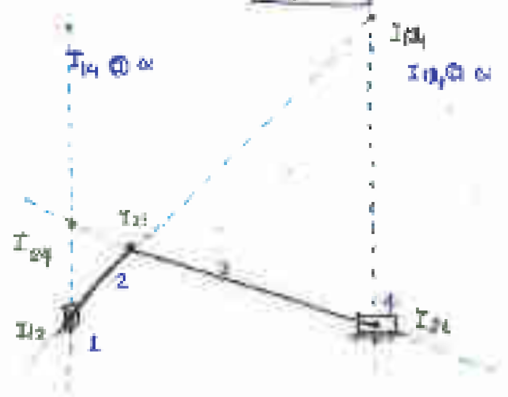
$\frac{V_P}{\frac{3}{2}R} = \frac{V}{R}$

$IP' = \sin 30 = \frac{IP'}{R}$

$IP = \frac{R}{2} + R = \frac{3R}{2}$

→ If a simple four-bar mechanism input link is parallel to output link angular speed of coupler will always be zero.

I_{13} I_{13} @ ∞ ⇒ $\omega_3 = 0$



→ ω_2 (given)

V_4 (slider velocity)

$P_{24} = P_{42}$

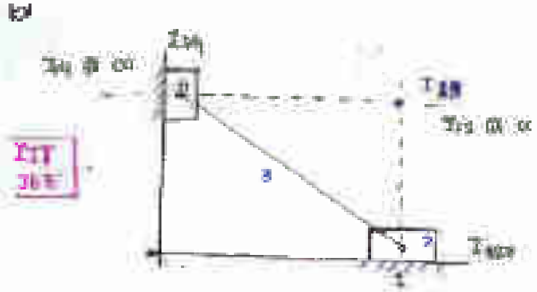
$V_{P_{24}} = V_{P_{42}}$

(link 4) (link 2)

$V_{slider} = (I_{12} - I_{24})\,\omega_2$

– In a single slider mechanism at an instant when crank is perpendicular to line of action I_{13} @ ∞ which leads $\omega_3 = 0$

(c)



I_{13} exists

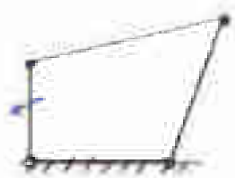
I_{24} @ ∞

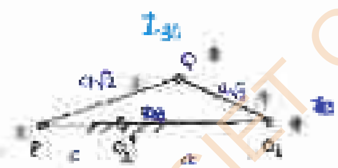
⇒ Rubbing velocity of pin

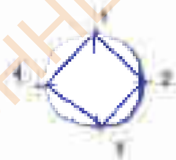
$$V_{pin} = r_{pin} (\omega_1 \pm \omega_2)$$

— If links are rotating in opposite direction (+ve)
same direction (−ve)

14







$\omega_2 (I_{12} I_{23}) = \omega_3 (I_{13} I_{23})$

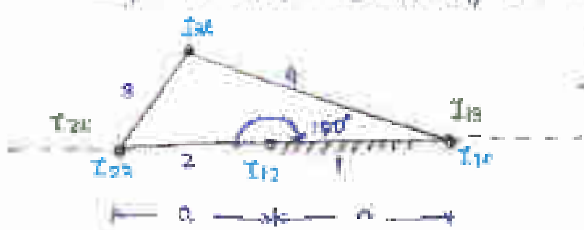
$\omega_3 = \frac{(1)(a)}{(2a)}$

$\boxed{\omega_3 = 1 \text{ rad/s}}$

Dehold / write

$\omega_4 (I_{14} I_{24}) = \omega_2 (I_{12} I_{24}) \Rightarrow \omega_4 = \frac{(1)(a)}{2a}$

$\boxed{\omega_4 = 1 \text{ rad/s}}$



(i) $V \cdot R = \frac{\omega_{i/p}}{\omega_{o/p}} = \frac{2}{1}$

$$\boxed{VR = \frac{2}{1}}$$

(ii) Deltoid linkage (or) kite linkage
Galloway linkage

(iii) $\gamma = 90°$ (transmission angle) [coupler ⊥ o/p link]

15

$\vec{V}_C = \vec{V}_O + \vec{V}_{C/O}$

$|\vec{V}_C| = OC \cdot \omega_2$ ⇒ $\boxed{\omega_2 = \frac{V_C}{OC}}$

$\omega_2 =$ (known)

$V_4 = (?)$

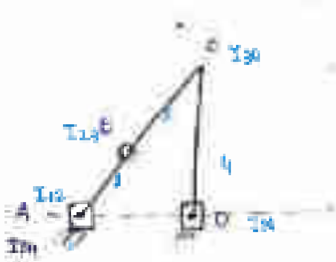
$\omega_2 (I_{12} I_{24}) = V_4$

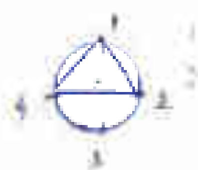
$\frac{OC}{\sin(90-\beta)} = \frac{OH (I_{12} \cdot I_{14})}{\sin(\alpha+\beta)}$

$I_{12} \cdot I_{14} = OC \cdot \sin(\alpha+\beta) \sec\beta$

$$\boxed{V_4 = V_C \sin(\alpha+\beta) \sec\beta}$$

16





$\omega_{DC} = (?) = \omega_4$

$\omega_4 (I_{14} I_{24}) = \omega_2 (I_{12} \cdot I_{24})$
$= 0$

→ In given problem pressure angle … corresponding to rocker will be at its extreme position (toggle position) so $\omega_{rocker} = 0$ [M.A = ∞]

[17]

$\omega_2 (I_{12} I_{24}) = \omega_4 (I_{14} I_{24})$

$\omega_2 (?) = \omega_4 (2?)$

$\boxed{\omega_4 = 1 \ rad/s}$

$\boxed{\omega_2 = 2 \ rad/s}$

$V_{rubbing\ velocity} = r_p (\omega_1 + \omega_2)$

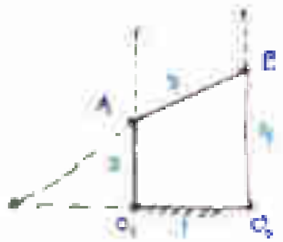
$V_1 = r_A (\omega_1 + \omega_2) = 10(0+2) = 20$

$V_2 = r_B (\omega_2 + \omega_3) = 10(2+0) = 20$

$V_3 = r_C (\omega_3 + \omega_4) = 10(0+1) = 10$

$V_4 = r_D (\omega_1 + \omega_4) = 10(0+1) = 10$

[Q] [17] Alternative approach



$\vec{V_A} = \vec{V_{O_1}} + \vec{V_{A/O_1}}$

$\boxed{|\vec{V_A}| = O_1A \ \omega_2}$

$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$

$= \vec{V_A} + (\omega_3 \times AB)$

$\boxed{\vec{V_B} = \vec{V_A}}$ $\quad |\omega_3 = 0$

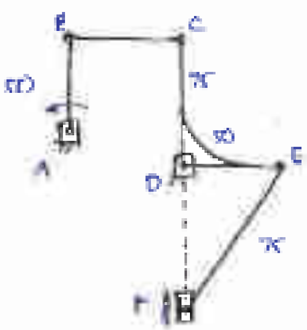
$\vec{V_B} = \vec{V_{O_2}} + \vec{V_{B/O_2}}$

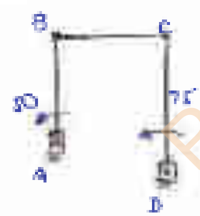
$\boxed{V_B = O_2B \cdot \omega_4}$

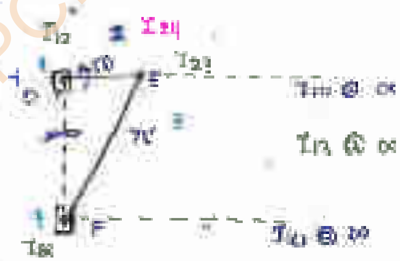
$O_1A \ \omega_2 = O_2B \ \omega_4$

$\boxed{l_{in} \ \omega_{in} = l_{out} \ \omega_{out}}$ ← for parallel links

[18]







AB ∥ CD

$l_{in} \ \omega_{in} = l_{out} \ \omega_{out}$

$50 \times 3 = \omega \times 75$

$\boxed{\omega_{out} = 2 \ rad/s}$

→ $\omega_{CD} = 2 \ rad/s \ (ccw)$

→ $\omega_{DE} = 2 \ rad/s \ (cw)$

$\omega_2 (I_{12} \ I_{24}) = \omega_4 (I_{14} - I_{24})$

$(2)(50) = V_F$

$\boxed{V_F = 100 \ m/s}$

$\bar{V}_E = \bar{V}_D + \bar{V}_{E/D}$

$\boxed{\bar{V}_E = DE \cdot \omega_3}$

$\bar{V}_F = \bar{V}_E + \bar{V}_{F/E}$

$= \bar{V}_E + (\bar{\omega}_3 \times \bar{EF})$

$\boxed{\bar{V}_F = \bar{V}_E}$

Hence $|\bar{V}_F| = DE \cdot \omega_3$

$\boxed{V_{slider} = r_2 \omega_2}$ ← angular velocity of crank

[20]

$\boxed{\omega_3 = 0}$ → $\boxed{I_{13} \text{ @ } \infty}$

If I_{13} @ ∞ crank is $\perp^r$ to line of stroke.

$V_{slider} = r_2 \omega_2$

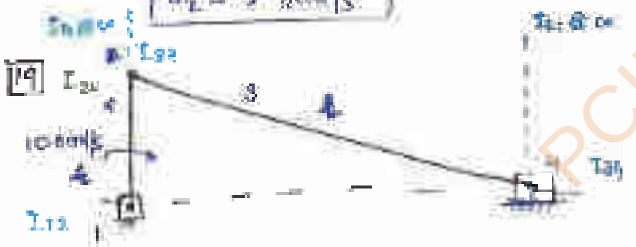
(ω) $\omega_2 = (r_2)$ [crank]

$V_{slider} = r_2 \omega_2$

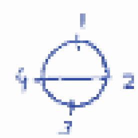
$8 = 4(1)\omega_2$

$\boxed{\omega_2 = 2 \text{ rad/s}}$

[19]



$\frac{\ell}{r} = 4$



$V_{slider} = 1$ m/s

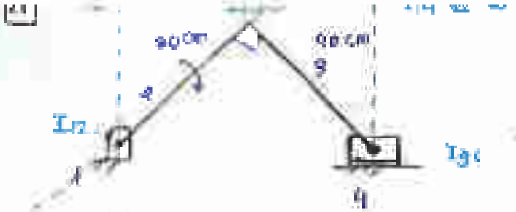
$\omega_4 (I_{14} \cdot I_{24}) = \omega_2 (I_{12} \cdot I_{24})$

$1 = 10 (I_{12} \cdot I_{24})$

$\boxed{r = 0.1}$

$\frac{\ell}{0.1} = 4$

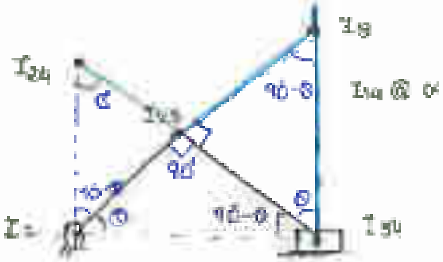
$\boxed{\ell = 0.4}$ C.R. length



$V_{slider} = ? (\omega_2 + \omega_3)$

$\omega_2 (I_{12} \cdot I_{23}) = \omega_3 (I_{13} \cdot I_{23})$

$(30)\ (90) = \omega_3 (58 \cdot 5)$



$$\frac{I_{13} \cdot I_{34}}{I_{13} \cdot I_{23}} = \frac{I_{12} \cdot I_{23}}{I_{14} \cdot I_{34}} = \frac{I_{12} \cdot I_{34}}{I_{14} \cdot I_{24}}$$

$$\frac{90}{I_{13} I_{23}} = \frac{30}{40}$$

$I_{13} I_{23} = 58 \cdot 5$ cm

$\omega_3 = 5 \cdot 62$ rad/s

→ $v_{pin} = 4 \cdot 5 (\omega_2 + \omega_3)$

$= 4 \cdot 5 (5 \cdot 625 + 10)$

$v_{pin} = 39$ m/s

→ for slider velocity

$\omega_4 (I_{14} \cdot I_{24}) = \omega_2 (I_{12} \cdot I_{24})$

$V_{slider} = \omega_2 (I_{12} \cdot I_{24})$

$V_{slider} = 376$ cm/s

$\tan\theta = \frac{90}{30}$

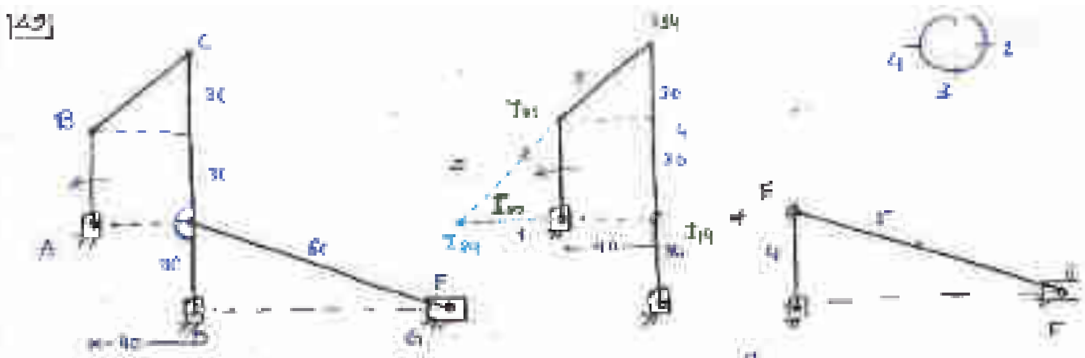
$\theta = 83 \cdot 12$

$\sin\theta = \frac{30}{I_{12} I_{24}}$

$I_{12} I_{24} = \frac{30}{\sin 83 \cdot 12}$

$I_{12} I_{24} = 37 \cdot 5$ cm

[49]



→ $\omega_2 = 6$ rad/s (given)

$\omega_4 = (?)$

$\omega_2(I_{12} - I_{24}) = \omega_4(I_{14} - I_{24})$

$\omega_2(40) = \omega_4(80)$

$\omega_4 = \frac{(6)(40)}{(80)}$

$\frac{60}{x+40} = \frac{30}{x}$

$2x = x + 40$

$x = 40$

1) Here two links are parallel so;

$l_{4} \omega_{4} = l_{6} \omega_{6}$

$\therefore (80)(6) = (40)\,\omega$

$\omega_6 = 2$ rad/s

2) $\omega_5 = (?)$

$\omega_4(I_{14} - I_{45}) = \omega_5(I_{15} - I_{45})$

I_{15} @ ∞

$\omega_5 = 0$

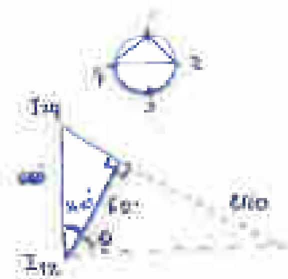
3) $V_{slider} = (?)$

$\omega_4(I_{14} I_{46}) = \omega_6(I_{16} I_{46})$

$(2)(30) = V_{slider}$

$V_{slider} = 60$ cm/s

[26]



$\omega_2(I_{12} - I_{24}) = \omega_4(I_{14} - I_{24})$

$10(61.94) = V_{slider}$

$V_{slider} = 619.4$ cm/s

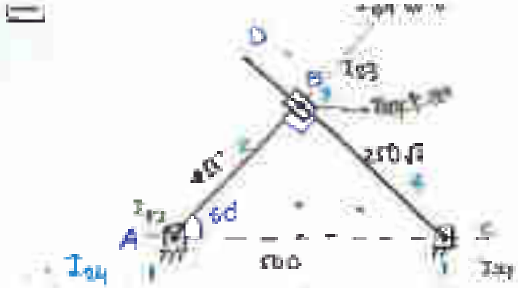
$= 6.19$ m/s

$\tan\theta = \frac{240}{60} = 4$

$\theta = 75.96$

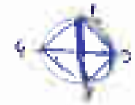
$\cos(14.03) = \frac{60}{I_{12} I_{24}}$

$I_{12} I_{24} = 61.84$



Observation

(i) crank + slotted bar mechanism
(ii) crank length is half of fixed link length
CRR = 2
(iii) ∠ABC = 90° so crank ⊥ to slotted bar
(iv) $\omega_{CD} = 0$ since it is at its extreme position (toggle)



→ $\omega_{AB} = 10$ rad/s (CCW)

$V_B = ?$

$$\frac{\omega_2}{\omega_4} = \frac{I_{12} \cdot I_{24}}{I_{12} \cdot I_{24}} \rightarrow (0)$$

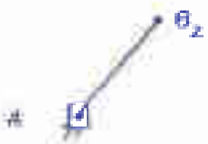
$\boxed{\omega_4 = 0}$

→ $V_{slider} = \omega_2 (I_{12} I_{23}) = \omega_3 (I_{13} I_{23})$

$= 10\,(250)$

$\boxed{V_{slider} = 2.5 \text{ m/s}}$

Alternative



→ $\vec{V}_{B_2} = \vec{V}_A + \vec{V}_{B_2/A}$

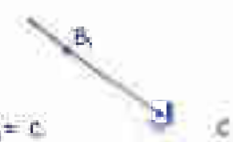
$= \vec{\omega}_2 \times \vec{AB}_2$

$\boxed{|\vec{V}_{B_2}| = AB_2 \cdot \omega_2}$

→ $\vec{V}_{B_3} = \vec{V}_{B_2} + \vec{V}_{B_3/B_2}$

$\boxed{|\vec{V}_{B_3}| = AB_2\,\omega_2}$

→ $\vec{V}_{B_3} = \vec{V}_{B_4} + \vec{V}_{B_3/B_4}$



$\vec{V}_{B_4} = \vec{V}_C + \vec{V}_{B_4/C}$

$= \vec{V}_C + \vec{\omega}_4 \times \vec{B_4C}$ $(\omega_4 = 0)$

$0 = \vec{V}_{B_4}$

$\vec{V}_{B_3} = \vec{V}_{B_3/B_4}$

$|\vec{V}_{B_3/B_4}| = AB \cdot \omega_2 = |\vec{V}_{B_3}|$

$= 250 \times 10$

$\boxed{|\vec{V}_{B_3/B_4}| = 2.5 \text{ m/s}}$

$CRR = \frac{1}{2} = \frac{\beta}{\alpha}$

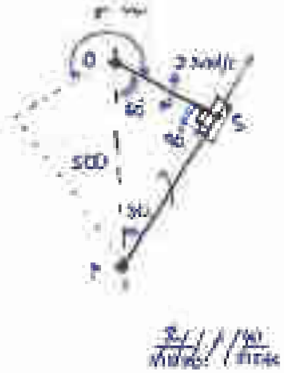
$\frac{\beta}{\alpha} = \frac{1}{2}$

$2\beta = \alpha$

$\alpha + \beta = 360$

$2\beta + \beta = 360$

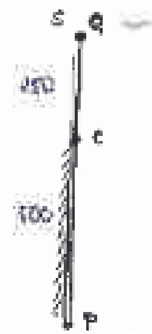
$\boxed{\beta = 120}$ & $\boxed{\alpha = 240}$



→ $\cos 60 = \frac{OS}{500}$

$\boxed{OS = 250 \text{ mm}}$

[7] max speed (ccw)



$\vec{V_c} = \vec{V_o} + \vec{V_{S/O}}$

$|\vec{V_c}| = |\vec{V_{S/O}}|$

$= OS \cdot \omega_2$

$= 250 \times 2$

$= 500$ mm/s

→ $\vec{V_q} = \vec{V_p} + \vec{V_{q/p}}$

$= PQ \cdot \omega_4$

$= 750\, \omega_4$

S & Q on same pt

$500 = 750\, \omega_4$ ⇒ $\boxed{\omega_4 = 2/3}$ rad/s

[8] velocity approach

$\boxed{\theta = 30 - 45}$

$\hat{OA} = \hat{i}\cos\theta + \hat{j}\sin\theta$

$\omega = 10$ rad/s (ccw)

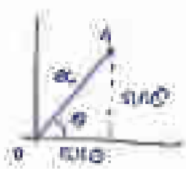
$\boxed{\vec{\omega} = 10\hat{k}}$

$\vec{V_A} = \vec{V_O} + \vec{V_{A/O}}$

$= \vec{\omega_2} \times \vec{AO}$

$= 10\hat{k} \times 80(\hat{i}\cos\theta + \hat{j}\sin\theta)$

$\boxed{\vec{V_A} = 800\cos\theta\,\hat{j} - 800\sin\theta\,\hat{i}}$



$\hat{OA} = \hat{i}\cos\theta + \hat{j}\sin\theta$

$\hat{AB} = \hat{i}\sin\theta - \hat{j}\cos\theta$

→ $\vec{V_B} = \vec{V_A} + \vec{V_{BA}}$

$= \vec{V_A} + (\vec{\omega_3} \times \vec{AB})$

$= \vec{V_A} + \omega_3\hat{k} \times 240(\hat{i}\sin\theta - \hat{j}\cos\theta)$

$\boxed{\vec{V_B} = \vec{V_A} + 240\,\omega_3\sin\theta\,\hat{j} + 240\,\omega_3\cos\theta\,\hat{i}}$

Let assume $\vec{\omega_3} = \omega_3\hat{k}$

$\Rightarrow V_B \hat{i} = 600\cos\theta \hat{j} - 600\sin\theta \hat{i} + 240\,\omega_3 \sin\theta \hat{j} + 240\,\omega_3 \cos\theta \hat{i}$

$V_B = 240\,\omega_3 \sin\theta - 600\sin\theta$

$0 = 600\cos\theta + 240\,\omega_3 \sin\theta$

$\omega_3 = -0.625$ rad/s (cw)

$V_B = -516.4$ mm/s (−x direction)

[34] $\omega_{CD} = 1$ rad/s (ccw) $= \omega_4$

$\omega_{AB} = ?$

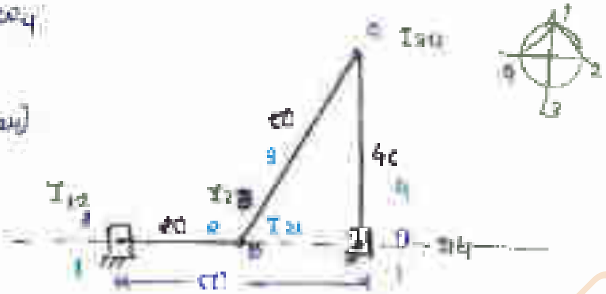
ω_4 ; $\omega_4 (I_{14}\, I_{24}) = \omega_2 (I_{12}\, I_{24})$

(1) ω_4 (30) = ω_2 (20)

~~ω_4 (ω_4 (30)) = (ω_2) (20)~~

(2) (30) = ω_2 (20)

$\omega_2 = 3$ rad/s



[35] $\omega_{AB} = 1$ rad (cw) $= \omega_2$

ω_{BC} , $\omega_{CD} = ?$

$\rightarrow \omega_2 (I_{12}\, I_{23}) = \omega_3 (I_{13}\, I_{23})$

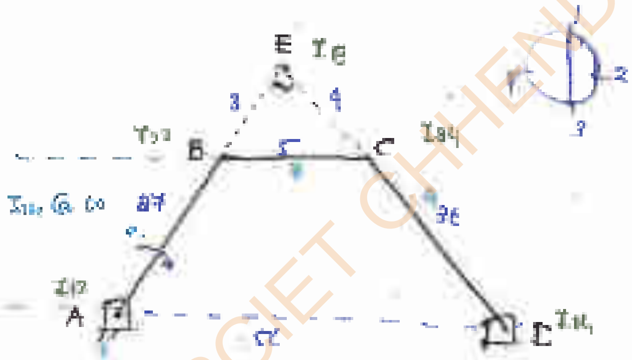
ω_2 (27) = ω_3 (3)

$\omega_3 = 9$ (ccw)

$\rightarrow \omega_3 (I_{13}\, I_{34}) = \omega_4 (I_{14}\, I_{34})$

(9) (4) = ω_4 (36)

$\omega_4 = 1$ rad/s (ccw)

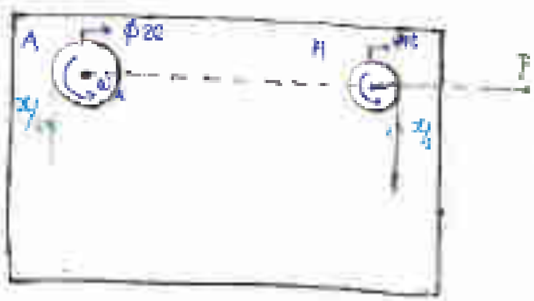


[36] the displacement of A in "x" that same at H

$V_A = V_H$

$10\,\omega_A = 5\,\omega_H$

$\frac{\omega_A}{\omega_H} = \frac{1}{2}$



$$\frac{\omega_8}{\omega_4} = \frac{HP}{AP} \rightarrow \omega_A(AH) = \omega_4(PH)$$

$$\frac{1}{2} = \frac{HP}{AP}$$

$$AP = 2\,HP$$

$$AH + HP = 2HP$$

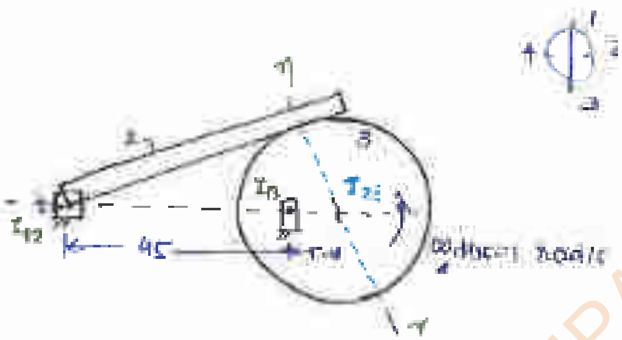
$$\boxed{AH = HP}$$

[39]

$$\omega_2(I_{12}\,I_{23}) = \omega_3(I_{13}\,I_{23})$$

(i) $\omega_2(150) = \omega_3(15)$

$$\boxed{\omega_3 = 0.1\ rad/s}$$



[40]

$AB = 1\ m$

$V_A = 2\ m/s$

$V_P = ?$

→ $\Delta IAP \cong \Delta IPB$

IP is comm^n

AP = PB & P is M.P

∠IAP = ∠IBP

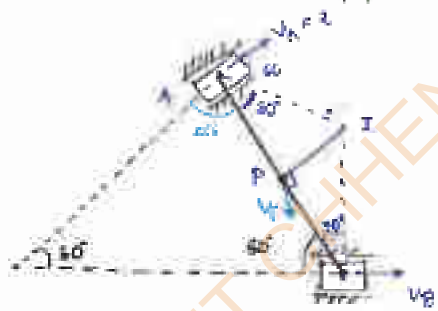
→ IP is ⊥^r to AB, V_P is along AB

$$\tan 30 = \frac{IP}{BP}$$

$$\boxed{IP = 0.5 \tan 30} = 0.288\ m$$

$$IB^2 = \sqrt{(IP)^2 + (BP)^2}$$

$$\boxed{IB = 0.577\ m = IA}$$



$$\frac{V_A}{IA} = \frac{V_P}{IP}$$

$$V_P = \frac{(2)(0.288)}{0.577}$$

$$\boxed{V_P = 1\ m/s}$$

Since : AB ∥ CD so

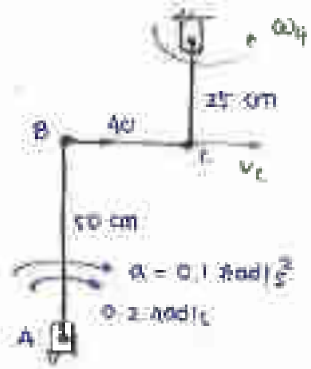
$\ell_{in}\,\omega_{in} = \ell_{out}\,\omega_{out}$

So, I_{BC} @ ∞ → $\omega_3 = 0$

→ $\ell_{in}\,\omega_{in} = \ell_{out}\,\omega_{out}$

$(50)(0.2) = (25)\,\omega_{out}$

$\omega_{out} = 0.4$ rad/s



for acc^n

→ $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$= 0 + \vec{a}_{n\,B/A} + \vec{a}_{t\,B/A}$

$= \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{AB}) + (\vec{\alpha}_2 \times \vec{AB})$

$= -0.2\hat{k} \times (-0.2\hat{k} \times 50\hat{j}) + (-0.1\hat{k} \times 50\hat{j})$

$= -0.2\hat{k} \times 10\hat{i} + 5\hat{i}$

$\vec{a}_B = -2\hat{j} + 5\hat{i}$

→ $\vec{a}_C = \vec{a}_B + \vec{a}_{C/B}$

$= \vec{a}_B + \vec{a}_{n\,C/B} + \vec{a}_{t\,C/B}$

$= \vec{a}_B + \vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{BC}) + \vec{\alpha}_3 \times \vec{BC}$

(0)

$= \vec{a}_B + \alpha_3\hat{k} \times 40\hat{i}$

$\vec{a}_C = \vec{a}_B + 40\alpha_3\hat{j}$

→ $\vec{a}_C = -2\hat{j} + 5\hat{i} + 40\alpha_3\hat{j}$

$\vec{a}_C = (40\alpha_3 - 2)\hat{j} + 5\hat{i}$ — ①

Assume,

$\vec{\alpha}_3 = \alpha_3\,\hat{k}$ (CCW)

$\vec{\alpha}_4 = \alpha_4\,\hat{k}$

→ $\vec{a}_C = \vec{a}_D + \vec{a}_{C/D}$

$= \vec{a}_{n\,C/D} + \vec{a}_{t\,C/D}$

$= \vec{\omega}_4 \times (\vec{\omega}_4 \times \vec{DC}) + \vec{\alpha}_4 \times \vec{DC}$

$= 0.4\hat{k} \times (0.4\hat{k} \times (-25)\hat{j}) + \alpha_4\hat{k} \times (-25\hat{j})$

$= 0.4\hat{k} \times 10\hat{i} + 25\alpha_4\hat{i}$

$\vec{a}_C = 4\hat{j} + 25\alpha_4\hat{i}$ — ②

eqn ① + ② $5 = 25\alpha_4$ ⇒ $\alpha_4 = 0.2$ rad/s² (CCW)

$40\alpha_3 - 2 = 4$ ⇒ $\alpha_3 = 0.15$ rad/s² (CCW)

→ If i/p link ∥ o/p link

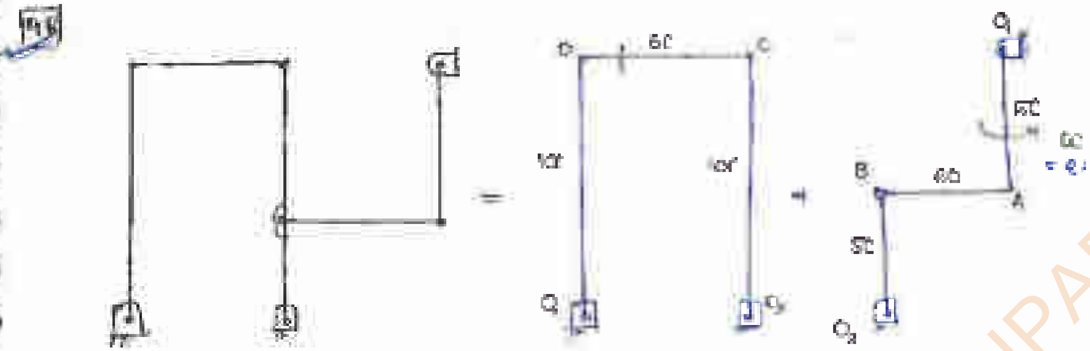
$\ell_{in}\,\omega_{in} = \ell_{out}\,\omega_{out}$

$\boxed{\ell_{in}\,\alpha_{in} = \ell_{out}\,\alpha_{out}}$

⇒ $50 \times \alpha_2 = 25 \times \alpha_4$

$50 \times 0.1 = 25 \times \alpha_4$

$\boxed{\alpha_4 = 0.2}$ rad/s²



In O_2ABO_3

→ $\ell_{in}\,\omega_{in} = \ell_{out}\,\omega_{out}$

$(50)(2) = (50)\,\omega_{out}$

$\boxed{\omega_{out} = \omega_{O_3B} = 2 \text{ rad/s}}$ ↑

→ $\alpha_{in} = 0$ (given)

$\ell_{in}\,\alpha_{in} = \ell_{out}\,\alpha_{out}$

$\boxed{\alpha_{out} = 0} = \alpha_{O_3B}$

In O_3CDO_4

$(O_3C)(\omega_{O_3C}) = (O_4D)(\omega_{O_4D})$

$(100)(2) = (100)(\omega_{O_4D})$

$\boxed{\omega_{O_4D} = 2 \text{ rad/s}}$

→ $O_3C\,(\alpha_{O_3C}) = O_4D\,(\alpha_{O_4D})$

$\boxed{\alpha_{O_4D} = 0}$

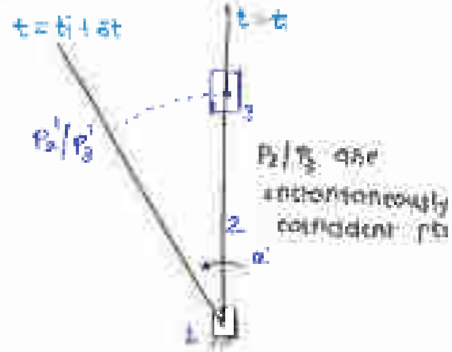
→ $\vec{V}_D = \vec{V}_{O_4} + \vec{V}_{D/O_4}$ ⇒ $|\vec{V}_D| = O_4D \cdot \omega_{O_4D}$

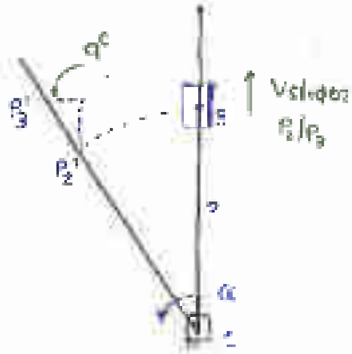
$= 100 \times 2$ ⇒ $\boxed{|\vec{V}_D| = 200 \text{ mm/s}}$

→ $\vec{a}_D = \vec{a}_{O_4} + \vec{a}_{D/O_4}$ ⇒ $|\vec{a}_D| = O_4D\,(\omega_{O_4D})^2 + (O_4D \cdot \alpha_{O_4D})$

$= 100 \times 2^2$

$\boxed{|\vec{a}_D| = 400 \text{ mm/s}^2}$





→ $\boxed{V_{P_3/P_1} = \text{zero}}$

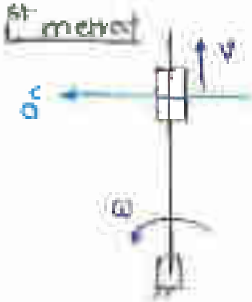
$$\boxed{\vec{a}_c = 2[\vec{\omega}_3 \times \vec{V}_{P_2/P_3}]}$$

⇒ Direction of coriolis acc^n

① Rotate the velocity vector by 90°.

② The sense of rotation should be same as ω

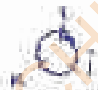
1st method

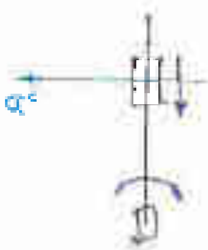


$\hat{a}^c$ = ω's direction × velocity's direction (acc. to right hand thumb rule)

$= \hat{k} \times \hat{j}$

$= -\hat{i}$

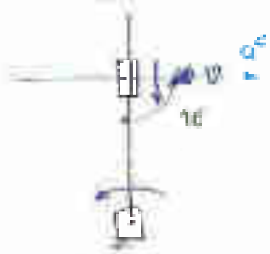




$\hat{a}_c = -\hat{k} \times -\hat{j}$

$= -\hat{i}$

2nd method



$\hat{a}_c = \hat{k} \times -\hat{j}$

$= \hat{i}$

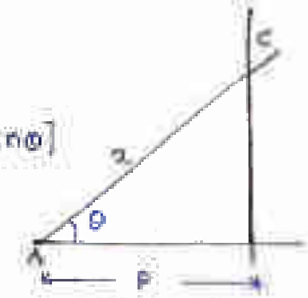
Rotate velocity vector by 90° in the same direction of 'ω' as that of

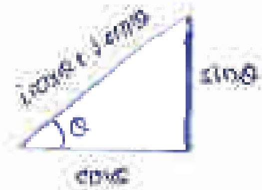
$\vec{AC} = |\vec{AC}| \cdot \hat{AC}$

$= x \cdot \hat{AC}$

$= \frac{p}{\cos\theta}[\hat{i}\cos\theta + \hat{j}\sin\theta]$

$\boxed{\vec{AC} = p[\hat{i} + \hat{j}\tan\theta]}$





$x\cos\theta = p$

$\boxed{x = \frac{p}{\cos\theta}}$

$\boxed{\frac{d}{dt}(\text{unit vector}) = 0}$

→ $\vec{v} = \frac{d\vec{AC}}{dt}$

$= \frac{d}{dt}[p\{\hat{i} + \hat{j}\tan\theta\}]$

$= p[0 + \hat{j}\sec^2\theta \frac{d\theta}{dt}]$

$\boxed{\vec{v} = \frac{p\omega}{\cos^2\theta}\hat{j}}$

→ $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\frac{p\omega}{\cos^2\theta}\right]$

$= p\omega \frac{d}{dt}[(\cos\theta)^{-2}]$

$= p\omega[(-2)(\cos\theta)^{-3} \cdot (-\sin\theta)\frac{d\theta}{dt}]$

$\boxed{\vec{a} = \frac{2p\omega^2\sin\theta}{\cos^3\theta}}$

[55] $l_{min} = 60$

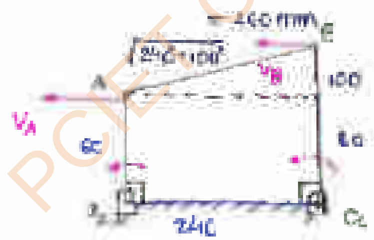
$l_{max} = 280$

$p = 240$

$q = 160$

$(60 + 280) < (240 + 160)$

→ crank rocker



$\tan(\theta) = \frac{100}{240}$

[56] For parallel links

$l_{in}\omega_{in} = l_{out}\omega_{out}$

$(60)(8) = (160)\,\omega_{out}$

$\boxed{\omega_{out} = 3\ \text{rad/s}}$

[illegible] For parallel $\boxed{\omega_3 = 0}$

[57]

{ Here ω = 0
α = 0
[illegible]
[illegible]
then [illegible] is
[illegible] }

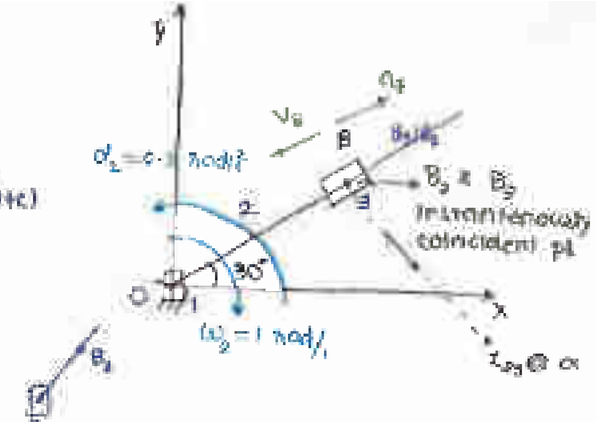
$|\vec{OB}| = 40$ cm

$\vec{OB} = \hat{i}\cos 30 + \hat{j}\sin 30$

$v_B = 0.2$ m/s

$a_B = 0.1$ m/s^2 (deceleration as opposite)

$\vec{a}_{B_3/O} = (?)$



$\rightarrow \vec{a}_{B_3} = \vec{a}_{B_2} + \vec{a}_{B_3/B_2} + \vec{a}_c$

$\rightarrow \vec{a}_{B_3} = \vec{a}_c + \vec{a}_{B_3/O}$

$\Rightarrow \vec{a}_{B_3} = \vec{a}_O + \vec{a}_{B_2/O} + \vec{a}_{B_3/B_2} + \vec{a}_c$

$= \vec{a}_{n\,B_2/O} + \vec{a}_{t\,B_2/O} + \vec{a}_{n\,B_3/B_2} + \vec{a}_{t\,B_3/B_2} + \vec{a}_c$

$= \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{OB}_2) + (\vec{\alpha}_2 \times \vec{OB}_2) + 0 + \vec{a}_B + 2(\vec{\omega}_2 \times \vec{v}_{B_3/B_2})$

$\{\vec{a}^n_{B_3/B_2} = 0$ as $\omega_3 = 0$

$\because \{ \vec{a}^n_{B_3/B_2} = \omega_3^2 r = \frac{v^2}{r}$

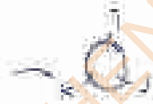
$\omega_3 = 0$ because $r = \infty$

Now:

$\vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{OB}_2) = -1\hat{k} \times (-1\hat{k} \times 40(\hat{i}\cos 30 + \hat{j}\sin 30))$

$= -1\hat{k} \times (-40\cos 30\,\hat{j} + 40\sin 30\,\hat{i})$

$= -40\cos 30\,\hat{i} - 40\sin 30\,\hat{j}$



$\vec{\alpha}_2 \times \vec{OB}_2 = 0.5\hat{k} \times 40(\hat{i}\cos 30 + \hat{j}\sin 30)$

$= 20\cos 30\,\hat{j} - 20\sin 30\,\hat{i}$

$\vec{a}_B = 50(\hat{i}\cos 30 + \hat{j}\sin 30)$

$\vec{a}_{B_3} = \vec{a}_{B_2/O} + \vec{a}_{B_3/B_2} + \vec{a}_B + \vec{a}_c$

$2(\vec{\omega}_2 \times \vec{v}_{B_3/B_2}) = 2[-1\hat{k} \times 20(-\hat{i}\cos 30 - \hat{j}\sin 30)]$

$= 40\cos 30\,\hat{j} - 40\sin 30\,\hat{i}$

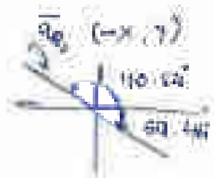
$\Rightarrow \vec{a}_{B_3} = -40\cos 30\,\hat{i} - 40\sin 30\,\hat{j} + 20\cos 30\,\hat{j} - 20\sin 30\,\hat{i}$
$+ 50\cos 30\,\hat{i} + 50\sin 30\,\hat{j} + 40\cos 30\,\hat{j} - 40\sin 30\,\hat{i}$

$= (-40\cos 30 - 20\sin 30 + 50\cos 30 - 40\sin 30)\,\hat{i}$
$+ (-40\sin 30 + 20\cos 30 + 50\sin 30 + 40\cos 30)\,\hat{j}$

$\boxed{\vec{a}_{B_3} = -21.339\,\hat{i} + 56.961\,\hat{j}}$

$\rightarrow |\vec{a}_{B_3}| = 60.82 = \underline{0.60\ \text{m/s}^2}$ ans.

$\theta = \tan^{-1}\left(\frac{56.961}{21.33}\right)$

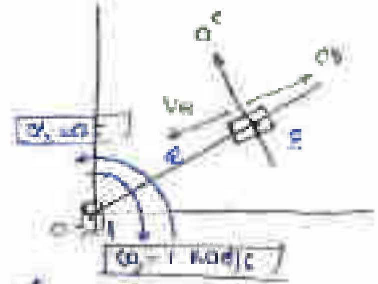


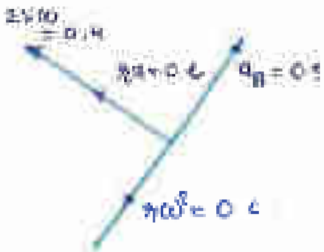
$\boxed{\theta_1 = -69.46°}$ & $\theta_2 = 180 - 69.46°$

$\boxed{\theta_2 = 110.54°}$

$$\vec a_{B_3} = \vec a^n_{B/O} + \vec a^t_{B/O} + \vec a_B + \vec a_C$$

$r\omega^2$ $\quad r\alpha \quad$ 0.5 $\quad 2V\omega$





$a_R = \sqrt{0.6^2 + 0.1^2}$

$a_R = 0.608$ m/s²

$\tan\theta = \frac{0.6}{0.1}$

$\theta = 80.53$

$\theta_{from\ x} = 80.53 + 30$

$\theta = 110.53$

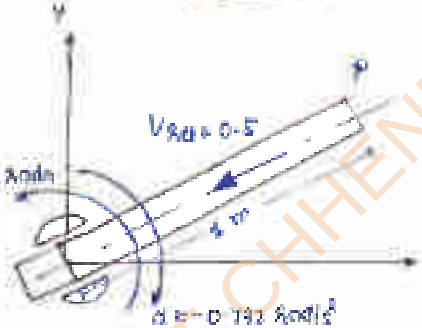
→ angle is taken from (+ve) x axis to resultant accⁿ

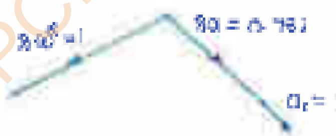
Q4

$$\vec a_{B_3} = \vec a^n_{B/O} + \vec a^t_{B/O} + \vec a_B + \vec a_C$$

(Radial accⁿ of arm w.r.t to base zero $\vec a_B = 0$)

$= r\omega^2 + r\alpha + 2\omega V$



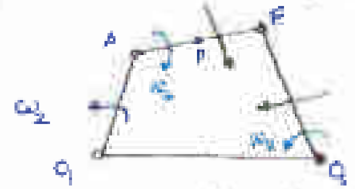


$\tan\theta = \frac{\sqrt3}{1}$

$\theta = 60$

$\theta = 270°$ from +x axis (to resultant)

Q. Simple four bar mechanism :-



Velocity Analysis

→ $\vec{V_A} = \vec{V}_{O_1} + \vec{V}_{A/O_1}$

$|\vec{V_A}| = O_1A \cdot \omega_2$
m, d, ⊥ to O₁A

→ $\vec{V_B} = \vec{V_A} + \vec{V}_{B/A}$

$= \vec{V_A} + \vec{\omega_3} \times \vec{AB}$
m d, m, d
✓ ✓, ×, ✓ ⊥ to AB

→ $\vec{V_B} = \vec{V}_{O_2} + \vec{V}_{B/O_2}$

$= \vec{\omega_4} \times \vec{O_2B}$
m, d --- ⊥ to O₂B

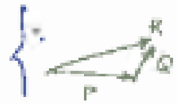
∴ $O_1a = \vec{V}_{A/O_1} = \vec{V_A}$

$ab = \vec{V}_{B/A}$

$|\vec{V}_{B/A}| = AB \cdot \omega_3$

$O_2b = \vec{V}_{B/O_2}$

$|\vec{V}_{B/O_2}| = (O_2B)\,\omega_4$



$\vec{R} = \vec{P} + \vec{Q}$

→ ✓ $\frac{AP}{AB} = \frac{ap}{ab}$ ✓

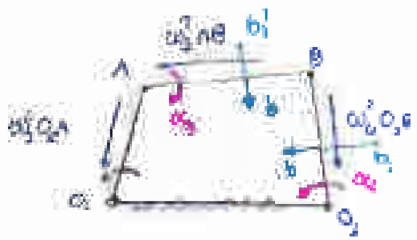
$ap =$ ___ (found)

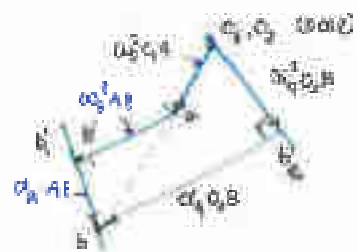
$\vec{V}_{P/A} = ap$

$\vec{V}_{P/B} = bp$

$\vec{V_P} = o_1p$

Acceleration Analysis :





→ $a_A = a_{O_1} + a_{A/O_1}$

$= \vec{a}^n_{A/O_1} + \vec{a}^t_{A/O_1}$

$= \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{O_1A}) + (\vec{\alpha}_1 \times \vec{O_1A})$

m ✓ d ✓ ⟶ $\parallel^{el}$ to link O_1A

→ $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$= \vec{a}_A + \vec{a}^n_{B/A} + \vec{a}^t_{B/A}$

$= \vec{a}_A + \vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{AB}) + (\vec{\alpha}_3 \times \vec{AB})$

m ✓ d ✓ ; m ✓ d ✓ ⟶ $\parallel^{el}$ to AB ; m ✗ d ✓ ⟶ $\perp^{el}$ to $\omega_3^2 AB$

→ $\vec{a}_B = \vec{a}_{O_2} + \vec{a}_{B/O_2}$

$= \vec{a}^n_{B/O_2} + \vec{a}^t_{B/O_2}$

$= \vec{\omega}_4 \times (\vec{\omega}_4 \times \vec{O_2B}) + (\vec{\alpha}_4 \times \vec{O_2B})$

m ✓ d ✓ ⟶ $\parallel^{el}$ to O_2B ; m ✗ d ✓ ⟶ $\perp^{r}$ to $\omega_4^2 O_2B$

⇒ $\vec{a}_{B/A} = ab$

$\vec{a}_B = ob$

★ Single Slider Mechanism :

(Velocity dia.)

→ $\vec{v}_A = \vec{v}_O + \vec{v}_{A/O}$

$= \vec{OA} \times \vec{\omega}_2$

m ✓ d ✓ ⟶ $\perp^{r}$ to OA

→ $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$= \vec{v}_A + \vec{\omega}_3 \times \vec{AB}$

m ✗ d ✓ ⟶ $\perp^{r}$ to AB

Accⁿ: $\vec{a}_A = \vec{a}_O + \vec{a}_{A/O} = \vec{a}^n_{A/O} + \vec{a}^t_{A/O}$

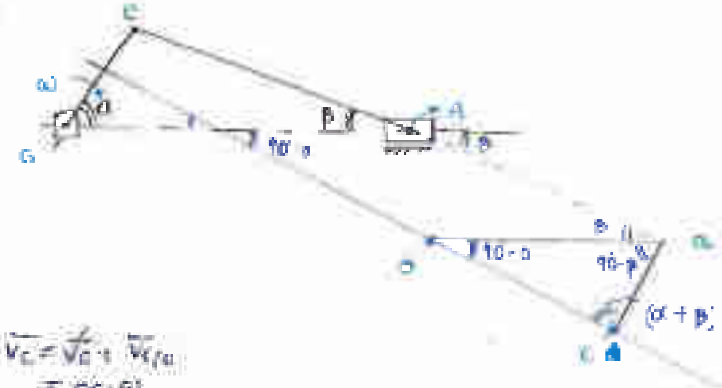
$= \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{OA}) + (\vec{\alpha}_2 \times \vec{OA})$

m ✓ d ✓ ⟶ $\parallel^{el}$ to OA

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \vec{a}^n_{B/A} + \vec{a}^t_{B/A}$

$= \vec{a}_A + \vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{AB}) + (\vec{\alpha}_3 \times \vec{AB})$

⟶ $\perp^{r}$ to $AB\,\omega_3^2$



$\vec{V_C} = \vec{V_O} + \vec{V_{C/O}}$

$= OC \cdot \omega_2$

$\& \; V_A = \vec{V_C} + \vec{V_{A/C}}$

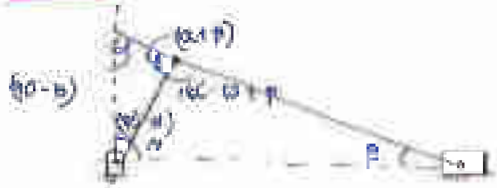
$= V_C + AC \cdot \omega_3$

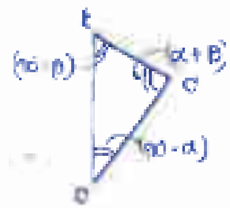
apply sine Rule

$\frac{OA}{\sin(\alpha+\beta)} = \frac{OC}{\sin(90-\beta)}$

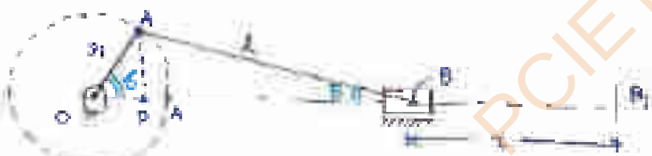
$V_A = V_C \sin(\alpha+\beta) \sec\beta$

Instead





⇒ Velocity and Acceleration analysis of slider crank mechanism (Analytic method)



$n = l/r$

Displacement of slider $= x = B_1B$

$= B_1O - BO$

$= (r+l) - (OP + PB)$

$= (r+l) - (r\cos\theta + l\cos\beta)$

$= (r + nr) - (r\cos\theta + nr\cos\beta)$

$= r[(n+1) - (n\cos\beta + \cos\theta)]$

But $n = \frac{l}{r}$

$r\sin\theta = l\sin\beta$ (AP)

$\sin\theta = n\sin\beta$

$\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{\sin^2\theta}{n^2}}$

$= \sqrt{\frac{n^2-\sin^2\theta}{n^2}} = \frac{\sqrt{n^2-\sin^2\theta}}{n}$

so Displacement $= x = r[(n+1) - (n\cdot\frac{\sqrt{n^2-\sin^2\theta}}{n} + \cos\theta)]$

Displacement of slider/piston $x = r[(1-\cos\theta) + (n - \sqrt{n^2-\sin^2\theta})]$

@ $\theta = 0$ $\quad x = 0$

$\theta = 180$ $\quad x = 2r$

Hence **stroke length = 2 x crank radius**

Special case: $n = 1$
i.e. $r = l$

Stroke length = 4r (practically not possible)

→ Velocity of slider:

$v = \frac{dx}{dt} = \frac{d}{dt}\{r[(1-\cos\theta) + (n - \sqrt{n^2-\sin^2\theta})]\}$

$= r\left[\frac{d}{dt}(1-\cos\theta) + \frac{d}{dt}(n - \sqrt{n^2-\sin^2\theta})\right]$

$= r\left[0 - (-\sin\theta)\frac{d\theta}{dt} + \{0 - \frac{1}{2}(n^2-\sin^2\theta)^{-1/2}(0 - 2\sin\theta\cos\theta\frac{d\theta}{dt})\}\right]$

$= r\left[\sin\theta\cdot\omega + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\cdot\omega\right]$

$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$

neglecting $\sin^2\theta$ ...

$V_{approx} = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$

@ $\theta = 90^\circ$ ⇒ $V_{slider} = r\omega$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[r\omega\left\{\sin\theta + \frac{\sin 2\theta}{2n}\right\}\right]$$

$$= r\omega\left[\cos\theta\frac{d\theta}{dt} + \frac{2\cos 2\theta}{2n}\cdot\frac{d\theta}{dt}\right]$$

$$= r\omega\left[\cos\theta\ \omega + \frac{\cos 2\theta}{n}\ \omega\right] \quad \text{'}\omega\text{' const}$$

$$\boxed{a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]} \leftarrow \text{where } \omega\text{' const}$$

→ Angular velocity & Angular acceleration of connecting rod

GATE - 6

$$\sin\beta = \frac{\sin\theta}{n}$$

$$\frac{d\beta}{dt} = \omega_{CR}, \quad \frac{d\theta}{dt} = \omega_{crank}$$

$$\cos\beta\frac{d\beta}{dt} = \frac{1}{n}\cos\theta\cdot\frac{d\theta}{dt}$$

$$\Rightarrow \quad \omega_{CR} = \frac{\omega\cos\theta}{n\cos\beta}$$

$$\cos\beta = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$n\cos\beta = \sqrt{n^2 - \sin^2\theta}$$

$$\boxed{\omega_{CR} = \frac{\omega\cos\theta}{\sqrt{n^2 - \sin^2\theta}}}$$

→ angular acceleration of connecting rod :

$$\alpha_{CR} = \frac{d\omega_{CR}}{dt}$$

$$= \frac{d}{dt}\left[\frac{\omega\cos\theta}{\sqrt{n^2 - \sin^2\theta}}\right]$$

$$= \frac{d}{dt}\left[\omega\cos\theta\ (n^2 - \sin^2\theta)^{-1/2}\right]$$

$$= \frac{d}{dt}\ \omega\left[\cos\theta\left(-\frac{1}{2}\right)(n^2 - \sin^2\theta)^{-3/2}(0 - 2\sin\theta\cos\theta)\frac{d\theta}{dt} + (-\sin\theta)(n^2 - \sin^2\theta)^{-1/2}\frac{d\theta}{dt}\right]$$

$$= \omega^2\left[\frac{\sin 2\theta\cdot\cos\theta}{2(n^2 - \sin^2\theta)^{3/2}} - \frac{\sin\theta}{(n^2 - \sin^2\theta)^{1/2}}\right]$$

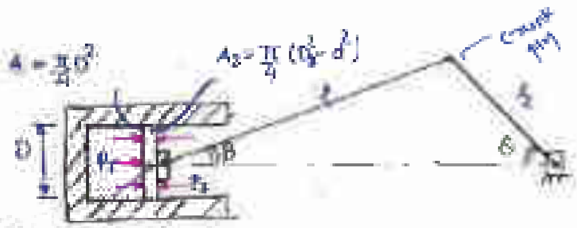
$$\boxed{\alpha_{CR} = \omega^2\left[\frac{\sin 2\theta\cdot\cos\theta}{2(n^2 - \sin^2\theta)^{3/2}} - \frac{\sin\theta}{(n^2 - \sin^2\theta)^{1/2}}\right]}$$

$\theta = 45°$
$\omega = const$

$$\alpha_{CR} = \frac{d}{dt}\left[\frac{\omega\cos\theta}{\sqrt{n^2 - \sin^2\theta}}\right]$$

$\boxed{\alpha_{CR} = C}$ ∴ ω const

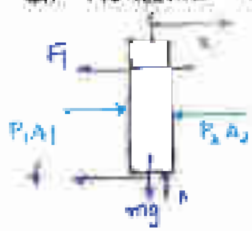
→ **Dynamic Force Analysis (in single slider Mechanism)**



where : m = mass of piston
Bore dia (or) = D
wrist pin dia = d
(or) C.R. dia
P_1 = pressure exerted by working substance
P_2 = pressure exerted by surrounding

(i) piston effort - net force acting on piston

In Horizontal Engine



$$F_{P_{net}} = (P_1A_1 - P_2A_2) - f - F_I$$
$$= (P_1A_1 - P_2A_2) - f - m a_{piston}$$

$$\boxed{F_{P_{net}} = (P_1A_1 - P_2A_2) - f - m r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]}$$

force in slides; friction force [illegible]

In Vertical Engine

$$\boxed{F_{P_{net}} = (P_1A_1 - P_2A_2) - f - m r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right] \pm mg}$$

where from TDC to BDC + mg
BDC to TDC − mg

NOTE :- Force in connecting Rod / thrust in gudgeon pin

→ Stroke = 2r
→ $(F_P)_{net}$ ≥ slider force

$F_{CR}\cos\beta = (F_P)_{net}$

$$\boxed{F_{CR} = \frac{(F_P)_{net}}{\cos\beta}}$$

(iii) Normal thrust b/w cylinder wall & piston :-

$$\boxed{N = F_{CR}\sin\beta}$$

(iv) Turning moment in the crankshaft $\boxed{\vec{T} = \vec{R}\times\vec{F}}$

$\vec{T} = \vec{R}\times\vec{F}$

$T = F_{CR}\sin\alpha \cdot R$

$$\boxed{T = \frac{F_P}{\cos\beta}\sin(\theta+\beta)\cdot R}$$

$\left\{ F_{CR} = \frac{F_{P\,net}}{\cos\beta} \right.$

(v) Thrust force in crank pin

$= F_{CR}\cos\alpha$

$$\boxed{F_{T_{crank\ pin}} = F_{CR}\cos(\theta+\beta)}$$

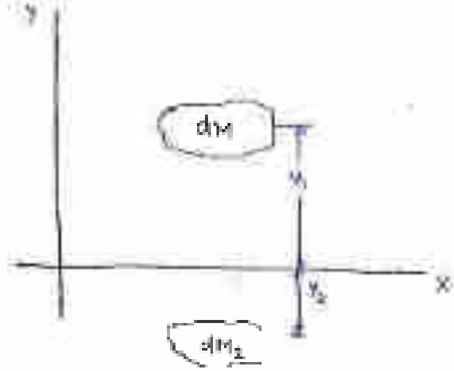
Note $T = f(\theta, \beta)$ but $\sin\beta = \frac{\sin\theta}{n}$

so $\boxed{T = f(\theta)}$

- Since the turning moment is function of θ (crank angle) & in order to run the engine we require uniform torque. Hence we require a device to make torque const. & name of device is flywheel.
- If it is ask to calculate θ at which force in gudgeon pin changes its direction then solve it by taking $\boxed{F_{P_{effect}} = 0}$ (for calculating gudgeon pin force)

piston effort = 0

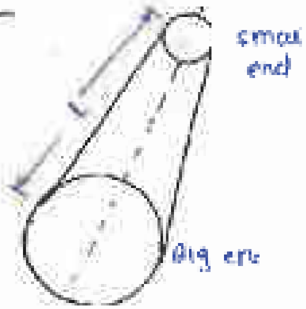
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots}$$



– second moment of mass $= y_1(y_1 \, dm_1)$ (dm_1)

$$dI = y_1^2 \, dm_1$$

–

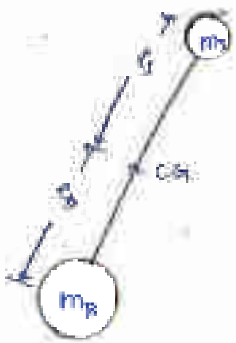


m = total mass of C.R.

m_s = mass @ small end

m_b " @ big end

$$m = m_s + m_b \quad \leftarrow (1)$$



$m_s l_s = m_b l_b$

$$m_s = m_b \frac{l_b}{l_s}$$

– $m = m_s + m_b = m_b + m_b \frac{l_b}{l_s}$

$$m_b = \frac{m \, l_s}{l_b + l_s}$$

$$m_s = \frac{m \, l_b}{l_b + l_s}$$

– $I_{act} = mk^2$

– $I_{eq^n \, system} = m_b l_b^2 + m_s l_s^2$

$I_{error} = I_{act} - I_{eq^n \, sys}$

[Ex] $l = 100$ cm, $l_b = 40$ cm

$m = 100$ kg, $k = 80$ cm

$l_b + l_s = l \rightarrow \boxed{l_s = 60 \text{ cm}}$

– $m_b = \frac{m \, l_s}{l_s + l_b} = \frac{(100)(60)}{(60)+(40)} \Rightarrow \boxed{m_b = 60 \text{ kg}}$

– $m_s = \frac{m \, l_b}{l_s + l_b} = \frac{(100)(40)}{(60)+(40)} \Rightarrow \boxed{m_s = 40 \text{ kg}}$

– $I = m_b l_b^2 + m_s l_s^2 = (40)(0.4)^2 + (60)(0.6)^2$

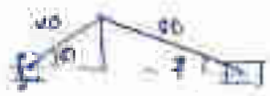
$= 24 \text{ kg·m}^2$

$r = 40$ cm
$l = 80$ cm

$\rightarrow F_T / F_{CR} = \dfrac{F \cos(\theta+\phi)}{\sin\beta}$

$n = \dfrac{l}{r} = 4$

$\sin\beta = \dfrac{\sin\theta}{n} = \dfrac{1}{4} = \boxed{14.48 = \beta}$



$F_{T(CR)} = \dfrac{(F_P)_{net}}{\cos\beta}$

at mid of stroke $\Rightarrow$ $\boxed{\theta = 90^\circ}$

$\rightarrow F_{T(CR)} = \dfrac{2\ kN}{\cos(14.50)}$

$\boxed{F_{T(CR)} = 2.065\ kN}$

$\rightarrow$ Twisting moment

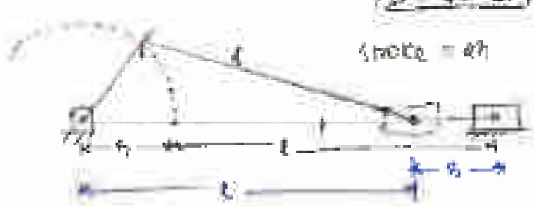
$T = \dfrac{(F_P)_{net}}{\cos\beta} \sin(\theta+\beta) \cdot r$

$= \dfrac{2\ kN}{\cos(14.50)} \sin(90 + 14.77) \times 0.2$

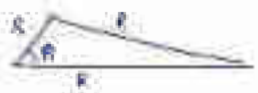
$\boxed{T = 6.4\ kN \cdot m}$

Analysis : when piston is at middle of stroke length

$\boxed{\theta = 62.5^\circ}$



$\theta = 90^\circ$ is not position $\rightarrow$ $l^2 = r^2 + l^2$?



$l^2 = r^2 + l^2 - 2rl\cos\theta$

$r^2 = 2rl\cos\theta$

$\cos\theta = \dfrac{r}{2l}$

$\theta = \cos^{-1}\left[\dfrac{r}{2l}\right]$

$\boxed{\theta = 82.8^\circ}$

[58] $T = I\alpha$ $\quad$ ①) average increasing $\rightarrow 0, \pi, 2\pi, 3\pi \rightarrow$ ⑥ ?

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

at t ever = 0 $\quad \theta = \frac{1}{2}\alpha t^2$

$\leftarrow$ [illegible] operation not [illegible]

$r = 900$ mm
$n = 600$
$\theta = 90^\circ$
$F_s = 5$ kN $= (F_P)$

$\rightarrow T = \frac{(F_P)_{net}}{\cos\phi} \sin(\theta + \phi) \cdot r$

$= \frac{5}{\cos\phi} \sin(90 + \phi) \cdot (0.2)$

$\boxed{T = 1 \text{ kN-m}}$

$\sin\phi = \frac{\sin\theta}{n}$

$= \frac{1}{4} \Rightarrow \boxed{\phi = 14.7^\circ}$

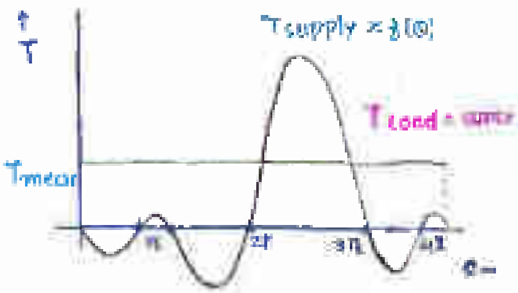
Energy Reservoirs

- Energy Reservoirs
  - Thermal Reservoirs [TER]
    - Source
    - Sink
  - Mechanical Energy Reservoirs [MER]
    - Ex flywheel

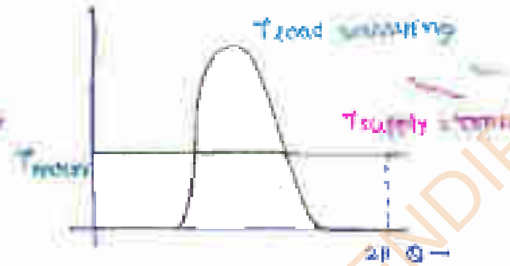
Rot. K.E $= \frac{1}{2} I \omega^2$

→ **Uses of flywheel:**

- Uses
  - In Engine
  - In punching press



4-stroke single cylinder I.C engine



Flywheel in engines

(i) $T_{act} > T_m$

$T_e = T_{act} - T_m$

flywheel will store energy

$\left(\frac{1}{2} I \omega^2\right) \uparrow$

$\omega \uparrow$

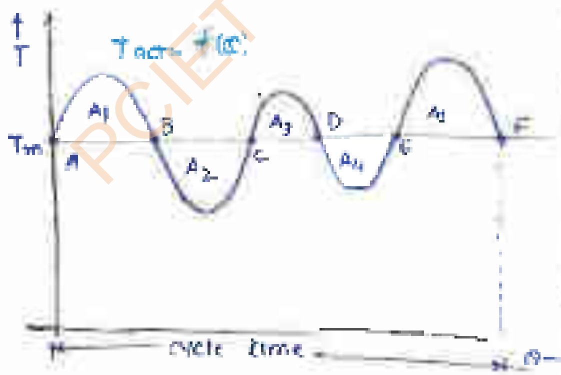
flywheel accelerating

(ii) $T_{act} < T_m$

flywheel will supply energy

$\omega \downarrow$

flywheel decelerating



⇒ $B = E_B = E_A +$ area of T vs θ dia betn A & B

$$\Rightarrow E_B = E_A + \int_{\theta_A}^{\theta_B} (T_{act} - T_m)\, d\theta$$

$$\boxed{E_B = E_A + A_1}$$

$$\Rightarrow \boxed{E_C = E_B - A_2}$$

$$E_C = E_A + A_1 - A_2$$

$$\Rightarrow \boxed{E_D = E_C + A_3}$$

$$= E_A + A_1 - A_2 + A_3$$

$$\Rightarrow \boxed{E_E = E_D - A_4}$$

$$= E_A + A_1 - A_2 + A_3 - A_4$$

$$\Rightarrow \boxed{E_F = E_E + A_5}$$

$$= E_A + A_1 - A_2 + A_3 - A_4 + A_5$$

— $\boxed{E_F = E_A}$

$$A_1 - A_2 + A_3 - A_4 + A_5 = 0$$

$$\boxed{A_1 + A_3 + A_5 = A_2 + A_4}$$

— let E_B is max

E_C is min

$E_{max} - E_{min}$ = max. fluctuation of $(\Delta K.E)_{max}$

$$= \frac{1}{2} I \omega_{max}^2 - \frac{1}{2} I \omega_{min}^2$$

$$\boxed{(\Delta K.E)_{max} = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)}$$

— $(\Delta K.E)_{max} = \frac{1}{2} I (\omega_{max} - \omega_{min})(\omega_{max} + \omega_{min})$

$$\boxed{(\Delta K.E)_{max} = I(\omega_{max} - \omega_{min})\, \omega_{mean}}$$

Here ω_{max} & ω_{min} close to each other so take arithmetic mean = ω_{mean}

— $(\Delta KE)_{max} = I(\omega_{mean})^2 \left(\frac{\omega_{max} - \omega_{min}}{\omega_{mean}}\right)$

$$\boxed{(\Delta K.E)_{max} = I \omega_m^2 C_s} \Rightarrow \boxed{\Delta E = 2 E C_s}$$

where, $C_s = \frac{\omega_{max} - \omega_{min}}{\omega_{mean}}$

C_s = co-efficient of fluction of speed

$C_s = \frac{\text{max. fluctuation of speed}}{\text{mean speed}}$

$\omega_{max} = 205$

$\omega_{mean} = 200$

$\omega_{min} = 195$

$\omega_{max} - \omega_{min} = C_s\,(\omega_{mean})$

$$\frac{N_{max} - N_{min}}{N_{mean}} = C_s$$

— If C_s value is small ⇒ fluctuation is small

NOTE:-
The prime job of flywheel is to reduce the speed fluctuation in a cycle.

	C_s
Pumps	$1/_{10} - 1/_{30}$
Reciprocating I.C. Eng.	$1/_{100} - 1/_{120}$
Aircraft	$1/_{1000}$

→ fluctuation → vibration → Dynamic loading
→ strength (↓) → failure (↑)

'C_s' should be small

$(\Delta K.E)_{max} = I\omega_m^2 C_s$

$I C_s$ = const

$$C_s \propto \frac{1}{I}$$

$I = mR^2$ (Ring or flywheel)

$I = \frac{mR^2}{2}$ (Disc flywheel)

→ If nothing is mentioned in problem take ring flywheel
$I = mR^2$ — because it having less fluctuation of speed compared to disc.

[*] Co-efficient of fluctuation of energy (C_E)

→ $C_E = \frac{\text{max. fluctuation of energy}}{\text{work done / cycle}} = \frac{(\Delta KE)_{max}}{WD/cycle}$

Work done / cycle = net area of T vs θ diag

— W.D/cycle = net energy Req^n per cycle

W.D/cycle = T_{mean} × cycle time

Two stroke engine → [cycle time = 2π]

Resisting torque is const T_m = const   N = 1000 rpm, P = ?

$P = \int T d\theta = \int 10000$

$W = \int T d\theta = \int_0^{2\pi} (10000 + 1000 \sin 2\theta - 1200 \cos 2\theta)\, d\theta$

$= 10000(2\pi) + 1000 \left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} - 1200 \left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}$

$= 20000\pi - 500[\cos 4\pi - \cos 0] - 600[\sin 4\pi - \sin 0]$

[W.D/cycle = 62800 J]

W.D / cycle = 10000 (2π) = $T_{mean} \times 2\pi$

[T_{mean} = 10000 N·m]

⇒ $P = \frac{2\pi \times 10000}{60}$ ⇒ [P = 104.7 kW]

→ Any eq^n of T the first term [ ] indicates the [T_{mean}]

→ Two stroke Engine → cycle time = 2π

Four stroke Engine → cycle time = 4π

[1c]

W.D/cycle = net area of +ve & -ve area

= $3000 \times \frac{\pi}{2} - 1500 \times \pi$

[W.D/cycle = 0]

W.D/cycle = $T_m \times 2\pi = 0$

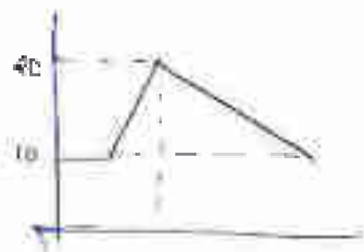
[T_m = 0]

→ E_A

→ $E_B = E_A + 1500\pi$ ← max

$E_C = E_B + 1500\pi - 1500\pi$

$= E_A$ ← min



$E_{max} - E_{min} = E_A + 1500\pi - E_A$

⇒ $\frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2) = 1500\pi$

⇒ $\frac{1}{2} I (40^2 - 10^2) = 1500\pi$ ⇒ [I = 6.28 kg·m²]

14 ...

$N_{avg} = 400$ RPM

$C_s = \pm 0.5\% = 1\%$

→ $(\Delta K.E) = I \omega_{mean}^2 C_s$

$2600 = I \left(\frac{2\pi \times 200}{60}\right)^2 \times \left(\frac{1}{100}\right)$

$2600 = I (4.36)$

$\boxed{I = 593.6 \ kg \cdot m^2}$

16

M → R: Ring, $I = mR^2$; M → R/2: disc, $I = \frac{mR^2}{2}$

→ M/2, R; M/2, R/2

$I = I_R + I_m$

$= mR^2 + \frac{mR^2}{2} = \frac{m}{2} R^2 + \frac{m}{2 \cdot 2}\left(\frac{R}{2}\right)^2$

$= \frac{mR^2}{2} + \frac{mR^2}{16}$

$\boxed{I = \frac{9}{16} mR^2}$

~~$I = I_R + I_M = mR^2 + \frac{mR^2}{2} = \frac{mR^2}{2} + \frac{mR^2}{4}$~~

~~$I = \frac{3}{4} mR^2$~~

15

$T = 400$ N·m

$N = 120$ RPM

$C_s = \pm 2\% = 0.04$

→ $(\Delta K.E) = I \omega_{mean}^2 C_s$

$400 = I \left(\frac{2\pi \times 120}{60}\right)^2 (0.04)$

$\boxed{I = 65 \ kg \ m^2}$

12

$C_s = \pm 2\% = 4\%$

$N_{mean} = 600$ rpm → $\omega_{mean} = 62.8$

$(K.E)_{max} = 5000$ J

$5000 = I (62.8)^2 \left(\frac{4}{100}\right)$ $\quad I \omega^2 C_s = E_m$

$I = 31.69$ kg m²

→ another flywheel attached same size $I_2 = 2I$

$C_s \propto \frac{1}{I} \Rightarrow \frac{C_{s2}}{C_{s1}} = \frac{I_1}{I_2} \Rightarrow \boxed{C_{s2} = 0.02}$

$R_1 = R_1 \rightarrow R_2 = 2R$

$M_1 = M_1 \rightarrow M_2 = M$

$\rightarrow (K.E) = I\omega_m^2 C_s$

$C_s \propto \frac{1}{I} \rightarrow \frac{C_{s_2}}{C_{s_1}} = \frac{I_1}{I_2} = \frac{\frac{1}{2} m R_1^2}{\frac{1}{2} m_2 R_2^2} = \frac{R_1^2}{(R_1)^2 4}$

$C_{s_2} = 0.01 \rightarrow 1\%$

10 E_A

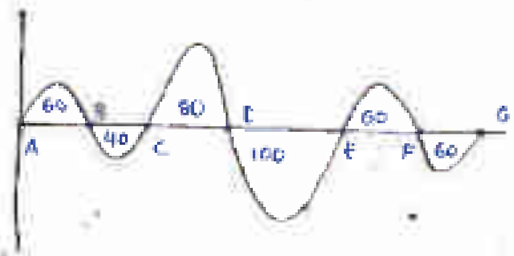
$E_B = E_A + 60$

$E_C = E_A + 60 - 40 = E_A + 20$

$E_D = E_A + 20 + 80 = E_A + 100$

$E_D > E_B > E_C > E_F$

R > P > Q > S

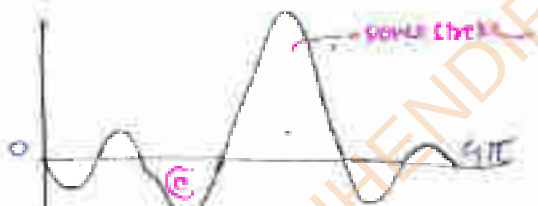


7

— If there is more than one power stroke than multi cylinder

— Power stroke – 720 – 4π

— comp stroke – 180 – 360



1 cm² = 100 N-m

$T_m \times 4\pi = [-0.5 + 1 - 2 + 2.5 - 0.5 + 0.5] 100$

$T_m = \frac{100}{\pi}$ N-m

8

$E_A = E_0 + 100$

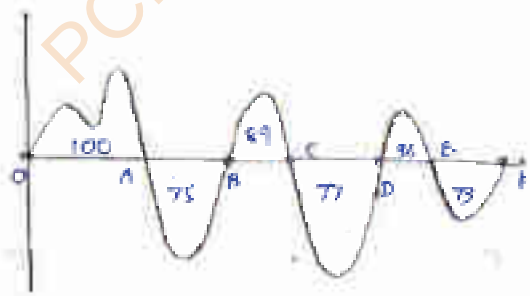
$E_B = E_0 + 25$

$E_C = E_0 + 119$

$E_D = E_0 + 37$

$E_E = E_0 + 73$

$E_F = E_0$ min



9 $N = 1200$ rpm, $I = I$, $C_s = 4\%$

$N_{avg} = \frac{N}{2}$, $C_s = 4\%$

$I_1 \omega_{mean_1}^2 C_{s_1} = I_2 \omega_{mean}^2 C_{s_2}$

$I_1 (1200)^2 (4\%) = I_2 (600)^2 (4\%)$

$I_2 = 4I_1$

$T_m \times 4\pi = \frac{1}{2} \times T_p \times \pi$

$10 \times 4\pi = T_p \times \frac{\pi}{2}$

$\boxed{T_p = 80\ N\ m}$

5

$k = 0.5$

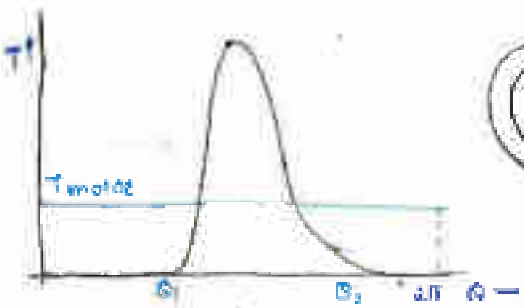
$N = 250\ rpm$

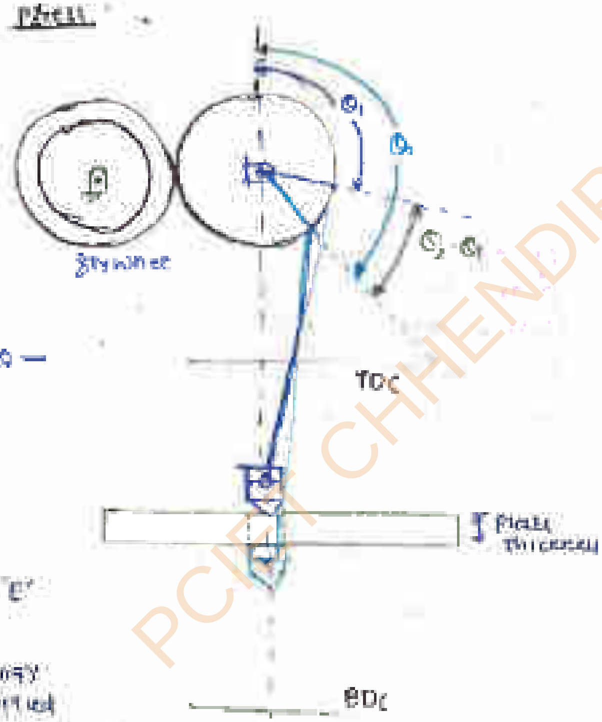
600 holes → per hr

$600 / (60 \times 60) = 0.166$ Holes/s

$t = 6\ s$

→ Flywheel in punching press.





Energy supply by motor / cycle

$\boxed{E_{motor} = T_m \times 2\pi}$

– let energy req^d per cycle 'E'

Energy req^d for punching = Energy consumed by motor during punching + Energy supplied by flywheel

$E_{flywheel}$ = Energy req^d during punching – energy req^d supplied by motor during punching

$2\pi \to E$

$1 \to E/2\pi$

$\theta_2 - \theta_1 \to \frac{E}{2\pi}(\theta_2 - \theta_1)$

$E_{flywheel} = E - \frac{E}{2\pi}(\theta_2 - \theta_1)$

$$E_{flywheel} = E\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right)$$

→ This valid for when idle stroke it not consume any energy

where $E_{flywheel} = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)$

$= I \omega_{mean}^2 C_s$

$$\frac{\theta_2 - \theta_1}{2\pi} = \frac{\text{thickness of plate}}{2 \times \text{stroke}} = \frac{\text{punching time}}{\text{cycle time}}$$

[5] $K = 0.5$ m (R.o.G)

$N_1 = 260$ rpm → $N_2 = 230$ rpm (speed should not drop so)

600 holes per hr {N_1 & N_2 is very close}

punching time = 1.5 sec

Energy req^n = 10,000 J

$P_{motor} = ?$ kW

$m = (?)$

→ $E_{flywheel} = E_{req^n} - E_{supply\ during\ punching}$

$= 10000 - 3000$

{ $P_{motor} = 2000$ J/s
1 sec → 2000 J
1.5 sec → ? }

$E_f = 7000$

→ $\frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2) = 7000$

$I \left(\frac{2\pi}{60}\right)^2 (260^2 - 230^2) = 14000$

$I = 86.93$ kg m^2

→ $I = mK^2$

$86.93 = m (0.5)^2$

$m = 347.7$ kg

[4] $d = 40$ mm, $t = 30$ mm ; $\tau_u = 7$ N m/mm^2 (shear area)

stroke = 100 mm , $t = 10$ sec → cycle time

$V_{mean} = 25$ m/s , $C_s = 8\% = 0.08$

$P_{motor} = ?$, F_d

→ sheared area $A = \pi d t = 3768$ mm^2

1 mm^2 → 7 N-m

3768 m^2 → (?)

$E_{req^n} = 26376$ J

by the motor in one cycle — (supplying energy is only assume)

In 10 sec : energy supplied by motor = 26376.0 J
1 sec " " " = 2637.6 J/s

$P_m = 2.63$ kW

$E_f = E_{req^d} -$ E_{supply} by motor during punching

$= 26376.0 - 2637.6$

$= 23738.4$ — plate thickness

$E_F = E\left[1 - \frac{\theta_2 - \theta_1}{2\pi}\right]$ check

$= E\left[1 - \frac{50}{2\times100}\right]$

$I\omega_m^2 C_s = 23419.6$ Joule — $I = mk^2$

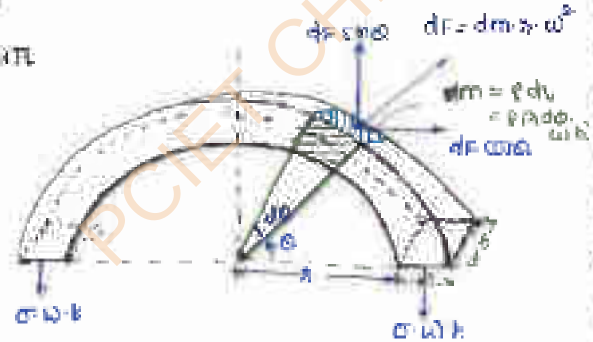
$\omega_m^2\, m(k^2)\, C_s = 23419.6$ — $r\theta = v$ — $V_m = k\omega$ — R.O.g = k = r

$m = \frac{23419.6}{(20)^2\times(0.03)}$

$m = 1195.71$ kg

[1] Four stroke cycle 4th = 4π

P = 80 kW
N = 300 rpm
C_s = 0.7 (?)
C_s = 0.02
σ_{max} = 6 MN/m²
d_m = (?)
ρ = 7200 kg/m³



$2\sigma wb = \int dF \sin\theta$

$2\sigma wb = \int dm\, r\,\omega^2 \sin\theta$

$= \int (r\,d\theta\, w\, b)\, \rho\, r\,\omega^2 \sin\theta$

$\sigma w = \int_0^{\pi} (\rho r\, d\theta)\, r\,\omega^2 \sin\theta\, d\theta$

$= \rho r^2\omega^2 \int_0^{\pi} \sin\theta\, d\theta$

$2\sigma = \rho v_m^2 [-\cos\theta]_0^{\pi} = \rho v_m^2 [-\cos\pi + \cos 0]$

$\sigma = \rho v_m^2$

$\rho = 7500\ kg/m^3$

$\sigma = \rho v_m^2$

$v_m = \sqrt{\frac{600000}{7500}} \Rightarrow \boxed{v_m = 28.28\ m/s}$

→ $v_m = \frac{\pi D_m N}{60} \Rightarrow D_m = \frac{28.28 \times 60}{\pi \times 300}$

$\boxed{D_m = 1.8\ m}$

→ $(K.E)_{max} = I \omega_m^2 C_s$

$C_E = \frac{(\Delta K.E)_{max}}{W.D/cycle}$

$W.D/cycle = T_m \times$ cycle time

$P = \frac{2\pi N T_m}{60}$

$T_m = \frac{60 \times 80 \times 10^3}{2\pi \times 300}$

$\boxed{T_m = 2546.47\ N\cdot m}$

→ $W.D/cycle = 2546.47 \times 4\pi$

$\boxed{W.D/cycle = 32000\ J}$

$C_E = \frac{(\Delta K.E)_{max}}{W.D/cycle}$

$(\Delta K.E)_{max} = 32000 \times 0.9$

$\boxed{(\Delta K.E)_{max} = 28800\ J} = I \omega_{mean}^2 C_s$

→ $28800 = I \left(\frac{2\pi \times 300}{60}\right)^2 \times 0.02$

$\boxed{I = 1459.02\ kg \cdot m^2}$

[2] Four stroke engine:

cycle time $= 4\pi$

$C_s = 0.01$

WD/cycle = net area of +ve @ dia

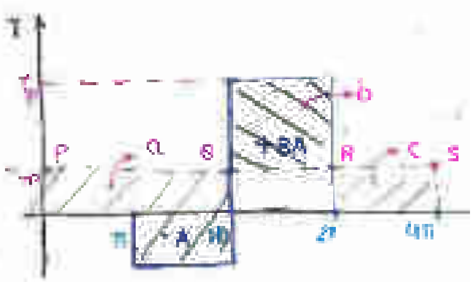
$= -A + 3A = 2A$

power = 20 kW

$N_m = 240$ rpm

$20 \times 10^3 = \frac{2\pi \times 240 \times T_m}{60}$

⇒ $\boxed{T_m = 795.7\ Nm}$



$T_m \times 4\pi = 2A$

$795.7 \times 4\pi = 2A$

$\boxed{A = 5000}$

→ $T_p \times \pi = 15000 = 3A$

$T_p = \frac{15000}{\pi}$

$\boxed{T_p = 4774.64 \text{ N·m}}$

~ E_P

$E_Q = E_P - a \Rightarrow E_{min}$

$E_R = E_P - a + b \Rightarrow E_{max}$

$E_S = E_P$

$\Delta E = E_R - E_Q$

$= b$

$b = (T_P - T_m)\pi$

$I\omega_m^2 C_s = (T_P - T_m)\pi$

$\boxed{I = 1976.5 \text{ kg m}^2}$

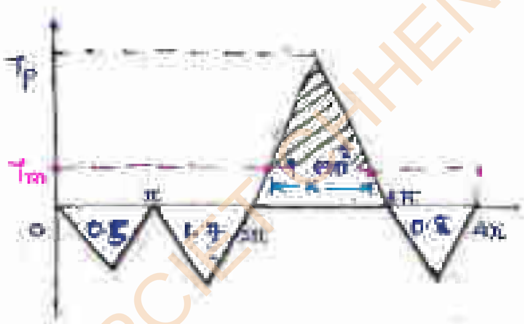
(3) $A = -0.5 + 1.7 + 4 - 0.8$

$= 0.4 + 4$

$= 4.4 \text{ cm}^2$ E

$\Rightarrow$ 1 cm² = 1400 J

6 cm² = 6400 J



WD/cycle = net area bet^n T vs θ dia.

$T_m \times 4\pi = 8400$

$\boxed{T_m = 668.48 \text{ N-m}}$

→ Expansion stroke = $\frac{1}{2} \times T_p \times \pi = 4 \times 1400$

$\boxed{T_p = 5025.47 \text{ N·m}}$

→ for energy fluctuation at C it is min & Hatched portion is max fluctuation area $(T_p - T_m)$ triangle

$\frac{x}{\pi} = \frac{T_p - T_m}{T_p} \Rightarrow \boxed{x = 2.769 \text{ rad}}$

$$\frac{1}{2} I(\omega_1^2 - \omega_2^2) = \frac{1}{2}(2.780)(6021.00 - 666.05)$$

$$I(\omega_1^2 - \omega_2^2) = 21147.11$$

$$I\left(\frac{2\pi}{60}\right)^2 (102^2 - 98^2) = 21147.11$$

$$\boxed{I = 2412.9 \text{ kg m}^2}$$

(13) Power of punching m/c = 2 kW (motor)

cycle time 1s = T

- punching stroke
- idle stroke

	punching stroke	idle stroke
cycle time	$\frac{T}{4}$	$\frac{3T}{4}$
Energy	$\frac{3}{4} \times (2T) = \frac{3}{2}T$	$\frac{1}{4} \times (2T) = \frac{T}{2}$

1 sec → 2 kJ
T sec → 2T
total energy = 2T

→ $C_E = \frac{\text{max}^n \text{ fluctuation of energy}}{\text{W.D / cycle}}$ → (flywheel energy)

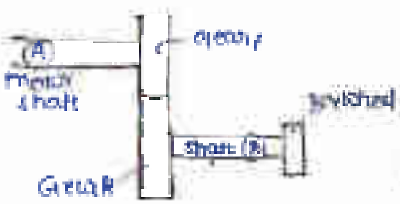
$E_{flywheel}$ = total energy req^n for punching − energy supplied by motor during punching

$$= \frac{3T}{2} - (2T)\left(\frac{T}{4}\right)$$

$$= T$$

$$C_E = \frac{T}{2T} = 0.5 \Rightarrow \boxed{C_E = 0.5}$$

(14)



reduction ratio = 4

$$\frac{\omega_A}{\omega_B} = \frac{4}{1} = \frac{n_B}{n_A}$$

→ Let by note :

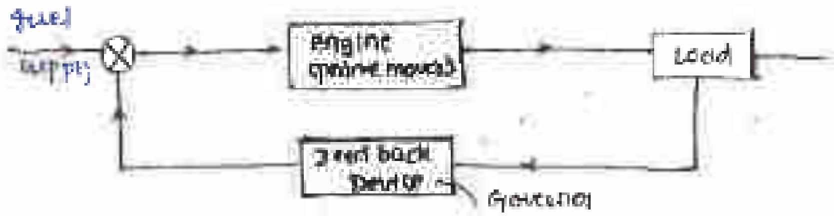
$$(\Delta K.E)_{max,B} = (\Delta K.E)_{max,A}$$

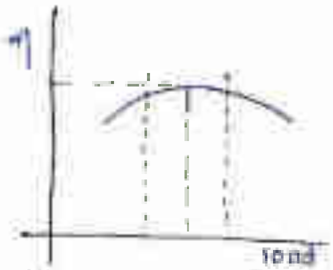
$$I(\omega_{m_B}^2) \cdot C_{s_1} = I(\omega_{m_A}^2) C_{s_2}$$

$$I(\omega_B)^2 (0.004) = I(\omega_A)^2 C_{s_2}$$

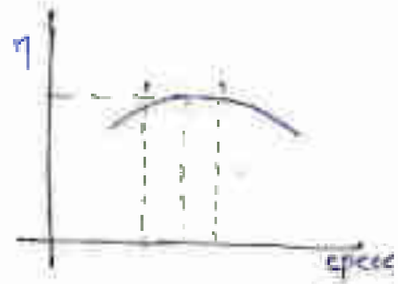
$$\boxed{C_{s_2} = 0.004} = \pm 4\%$$

$$= \pm 0.2\% \text{ (equation)}$$





(Load vs η)

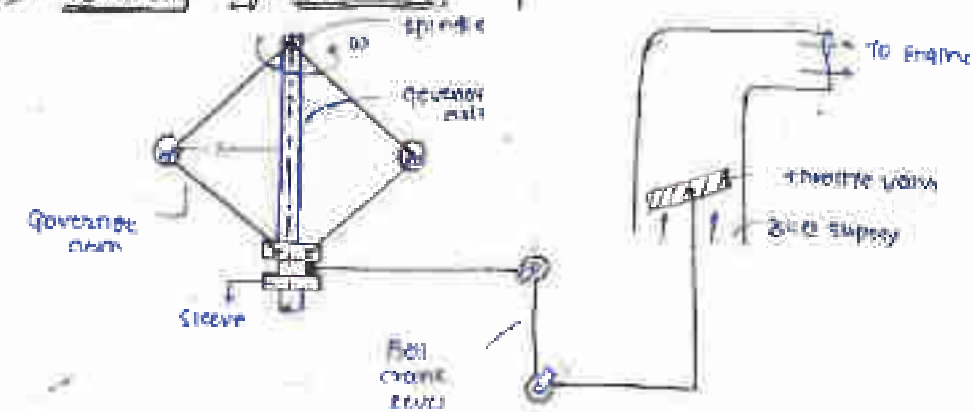


(η vs speed)

- Governors are feedback devices which regulates the fuel supply with respect to variation in load at output of primemover.

Flywheel	Governor
- Flywheel works continuously (each stroke)	- Governor works intermediately
- Flywheel regulates the speed fluctuation within a cycle.	- Governor regulates the speed fluctuation in between two cycle.
- Flywheel regulates each cycle fluctuation (within)	- Governor regulates entire cycle fluctuation (between).

⇒ Working of Governor

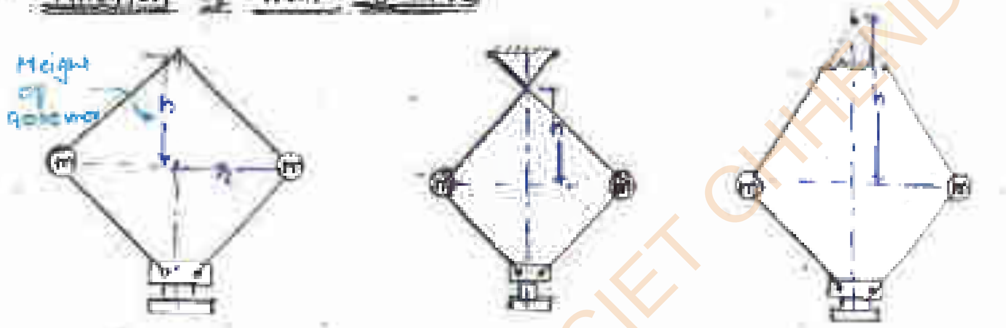


(a) ↓
Sleeve comes down
throttle valve open
fuel supply ↑

→ Classification of Governors:

Governors
- Centrifugal Governors
  - Pendulum type
    - Watt Governor
  - Gravity controlled
    - Porter
    - Proell
  - Spring controlled
    - Hartnell
    - Wilson Hartnell
    - Hartung
- Inertial Governors
  - Obsolete

→ Analysis of Watt Governor:



Simple watt governor

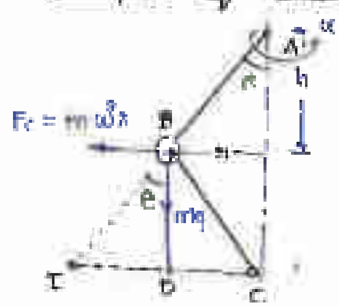
crossed arm watt

open arm type watt governor

Height of governor → Distance between plane carrying governor balls to the point where governor arms intersect with governor axis.

→ Analysis of watt governor

- sleeve mass negligible
- inertia of governor arms negligible



$\Sigma M_I = 0$

$\Rightarrow m\omega^2 r \, BD = mg \, ID$

$\Rightarrow \omega^2 r = g \left(\frac{ID}{BD}\right)$

$\Rightarrow \omega^2 r = g \tan\theta$

$\Rightarrow \boxed{\omega^2 = \frac{g}{h}}$

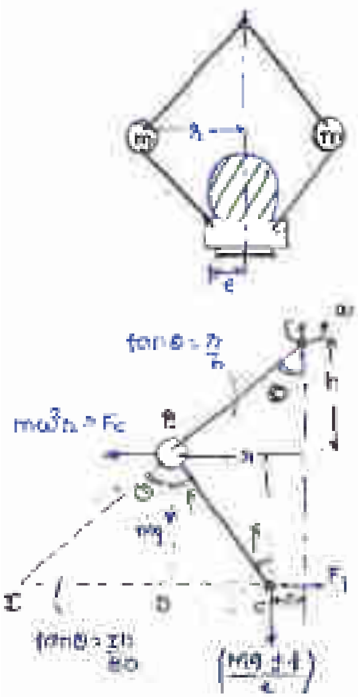
$\tan\theta = \frac{r}{h}$

$\rightarrow \omega^2 \propto \frac{1}{h}$; $\omega \propto \frac{1}{\sqrt{h}}$ $\rightarrow$ $\frac{\omega_1}{\omega_2} = \sqrt{\frac{h_2}{h_1}}$

$\rightarrow \left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}$

$h = \frac{895}{N^2}$ $\rightarrow$ $N^2 = \frac{895}{h}$

Q) Porter Governor



m = mass of ball
M = sleeve mass

→ friction of governor arms are negligible

$\Sigma M_I = 0$ (ACW +ve)

$\rightarrow m r \omega^2 (BD) - mg(DI) - \left(\frac{Mg \pm f}{2}\right) IC = 0$

$\Rightarrow m r \omega^2 (BD) = mg(DI) + \left(\frac{Mg \pm f}{2}\right) IC$

$= mg\left(\frac{DI}{BD}\right) + \left(\frac{Mg \pm f}{2}\right)\left(\frac{IC}{BD}\right)$

$= mg\left(\frac{DI}{BD}\right) + \left(\frac{Mg \pm f}{2}\right)\left(\frac{ID}{BD} + \frac{DC}{BD}\right)$

$m r \omega^2 = mg \tan\theta + \left(\frac{Mg \pm f}{2}\right)(\tan\beta + \tan\theta)$

$m\omega^2 r = \tan\theta \left[mg + \left(\frac{Mg \pm f}{2}\right)\left(1 + \frac{\tan\beta}{\tan\theta}\right)\right]$

$= \frac{r}{h}\left[mg + \left(\frac{Mg \pm f}{2}\right)(1+K)\right]$

$\left\{ K = \frac{\tan\beta}{\tan\theta} \right.$

$$\omega^2 = \frac{2mg + (Mg \pm f)(1+K)}{2mh}$$

where, $K = \frac{\tan\beta}{\tan\theta}$

Special case

If $K = 1$ i.e. $\theta = \beta$

i.e. Governor arms are equal length and both arms are pivoted on governor axis.

$2mh$

$$\omega^2 = \frac{mg + (Mg \pm f)}{mh}$$

→ 1) friction is neglected

$$\omega^2 = \left(\frac{m + M}{m}\right) \cdot \frac{g}{h}$$

(when friction neglected)

→ 2) sleeve mass neglected

$$\omega^2 = \frac{g}{h}$$

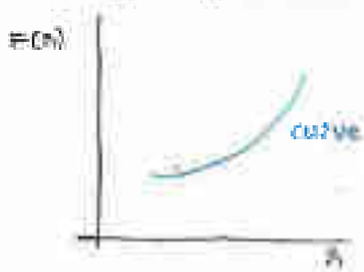
(when sleeve mass neglected) → Watt governor

Governor terminology

1] Controlling force / Resisting force

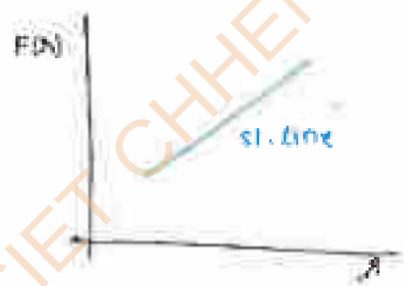
- The resultant of all frictional force that forces (i.e. centrifugal governor) & spring forces is known as controlling forces

For gravity controlled governor



Slope of controlling force curve $= \frac{dF(r)}{dr}$

For spring controlled gov.



→ Centrifugal force

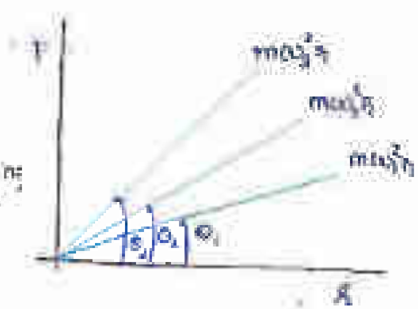
$$F = mr\omega^2$$

Slope of centrifugal force curve $= m\omega^2 = \frac{F}{r}$

$$\tan\theta_3 > \tan\theta_2 > \tan\theta_1$$

$$\omega_3 > \omega_2 > \omega_1$$

→ by increasing speed

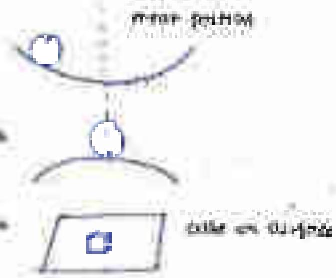


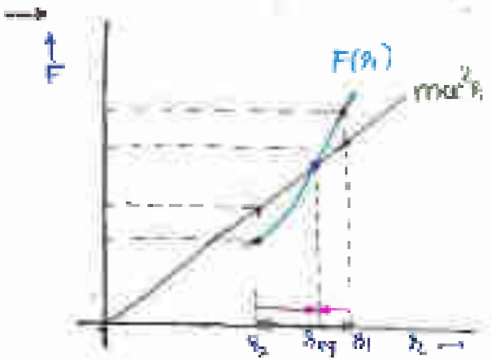
(xx) Types of equilibrium

a) stable eq^n →

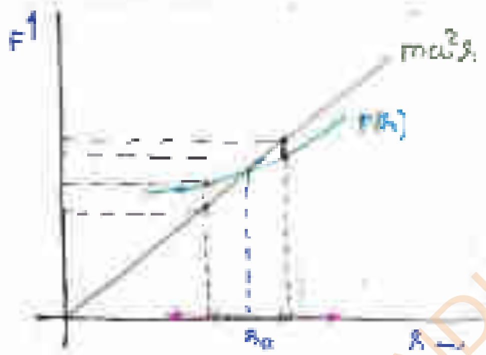
b) unstable eq^n →

c) Neutral eq^n →





Stable governor



Unstable governor

Stable governor:

slope of controlling force curve > slope of disturbing force

$$\rightarrow \frac{dF(r)}{dr} > \frac{F}{r}$$

$$\rightarrow \boxed{\frac{dF(r)}{dr} > m\omega^2}$$

Unstable governor:

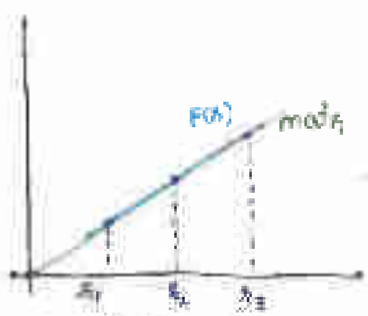
slope of controlling force < slope of disturbing force

$$\boxed{\frac{dF(r)}{dr} < \frac{F}{r}}$$

$$\boxed{\frac{dF(r)}{dr} < m\omega^2}$$

→ controlling force → decrease speed (↓)
disturbing force → increase

c) Neutral equation → (isochronous governor)



$r_1 < r_2 < r_3$

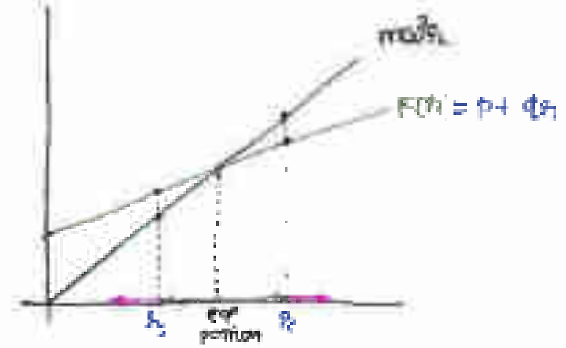
$\boxed{\omega_1 = \omega_2 = \omega_3 = \text{const.}}$

slope of controlling force curve = slope of disturbing force

$$\frac{dF(r)}{dr} = \frac{F}{r}$$

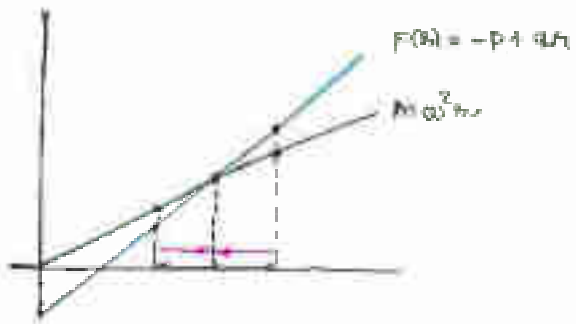
$$\boxed{\frac{dF(r)}{dr} = m\omega^2}$$

→ practically it is not possible



unstable eqn:
p & q are "+ve"
$p > 0$
$q > 0$

unstable



stable eqn:
$p < 0$; -ve
$q > 0$; +ve

→ Range:

Range of governor = $\omega_{max} - \omega_{min}$

Ex. Range of isochronous governor = 0

→ Sensitiveness:

- A ability of governor to sense the change in output load & move its sleeve displacement accordingly.
- If for some change in load sleeve displacement of governor A is more than that of governor B then governor is said to be more sensitive than B

→ Co-efficient of sensitiveness or sensitiveness of governor.

Co-efficient of sensitiveness = $\frac{\omega_{mean}}{\omega_1 - \omega_2}$ or sensitiveness ← GATE, IES, DEC

Ex. sensitiveness of isochronous governor is ∞

Co-efficient of sensitiveness = $\frac{\omega_1 - \omega_2}{\omega_{mean}}$ ← Clock Rom
(or) sensitiveness

- When a governor is connected to some prime mover at that it will take the function of the prime mover & its sensitivity will be $\frac{N_1 - N_2}{N_{mean}}$

If stiffness of governor is large then rather than working at some equilibrium position the sleeve will tend to oscillate that is vibrate about mean position, at this extent governor is said to be Hunting.

Effort of Governor:

- Mean force exerted on the sleeve is known as effort.

Porter governor

$$h = \frac{2mg + (Mg \pm f)(1+k)}{2m\omega^2} \quad \text{--- (i)}$$

let $\boxed{\omega_1 = (1+c)\,\omega}$ — fraction

$$\omega_1^2 = \frac{2mg + (Mg \pm f)(1+k)}{2mh_1}$$

if $\omega_1 > \omega$

sleeve moves up.

$$\omega_1^2 = \frac{2mg + (Mg \pm f + E)(1+k)}{2mh}$$

$$h = \frac{2mg + (Mg \pm f + E)(1+k)}{2m\omega_1^2} \quad \text{--- (ii)}$$

From eqⁿ (i) & (ii)

$$\Rightarrow \frac{2mg + (Mg \pm f)(1+k)}{2m\omega^2} = \frac{2mg + (Mg \pm f + E)(1+k)}{2m\omega_1^2}$$

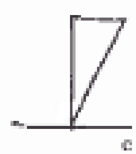
$$\Rightarrow \frac{2mg + (Mg \pm f)(1+k)}{2m\omega^2} = \frac{2mg + (Mg \pm f + E)(1+k)}{2m(1+c)^2\omega^2}$$

$$\Rightarrow [2mg + (Mg \pm f)(1+k)](1 + c^2 + 2c) = 2mg + (Mg \pm f + E)(1+k)$$

$$\Rightarrow 2mg + (Mg \pm f)(1+k) + 2c\,[2mg + (Mg \pm f)(1+k)] = 2mg + (Mg \pm f)(1+k) + E(1+k)$$

$$\boxed{E = \frac{2c}{(1+k)}\,[2mg + (Mg \pm f)(1+k)]}$$

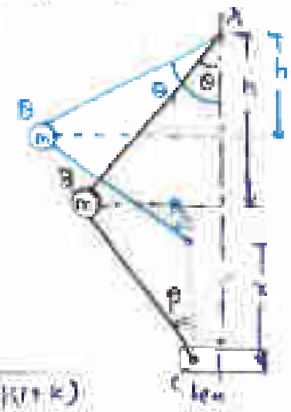
Effort of governor



E is a mean force which varies from 0 to maximum c.c. divide by 2.

→ the workdone by effort is known as power.

power = effort × sleeve displacement



$\therefore h = \frac{2mg + (Mg \pm f)(1+K)}{2m\omega^2}$

$\Rightarrow h_1 = \frac{2mg + (Mg \pm f)(1+K)}{2m\omega_1^2}$

$\Rightarrow h_1 = \frac{2mg + (Mg \pm f)(1+K)}{2m(1+c)^2\omega^2}$

$\therefore \frac{h_1}{h} = \frac{2mg + (Mg \pm f)(1+K)}{2m(1+c)^2\omega^2} \times \frac{2m\omega^2}{2mg + (Mg \pm f)(1+K)}$

$\Rightarrow \boxed{\frac{h_1}{h} = \frac{1}{(1+c)^2}}$

→ x = Displacement sleeve

$x = (AB\cos\theta + BC\cos\beta) - (AB_1\cos\theta_1 + B_1C_1\cos\beta_1)$

$= (h + BC\cos\beta) - (h_1 + B_1C_1\cos\beta_1)$

$x = (h - h_1) + (BC\cos\beta - B_1C_1\cos\beta_1)$

⇒ If θ = β (sleeve length same)

$x = 2AB\cos\theta - 2AB_1\cos\theta_1$

$\boxed{x = 2(h - h_1)}$ ← when θ = β (K = 1), sleeve displacement

→ $power = \frac{E}{2} \times x$

$\frac{E}{2} = \frac{c}{(1+K)}[2mg + (Mg \pm f)(1+K)]$

if K = 1, θ = β

⇒ $\frac{E}{2} = \frac{c}{2}[2mg + (Mg \pm f)(2)]$

$\boxed{\frac{E}{2} = c\,[mg + (Mg \pm f)]}$

⇒ $\boxed{power = \frac{E}{2} \times x}$

→ $power = \frac{E}{2} \times 2(h - h_1) = \frac{E}{2} \times 2h\left[1 - \frac{1}{(1+c)^2}\right]$ $\left\{ \frac{h_1}{h} = \frac{1}{(1+c)^2} \right.$

$= \frac{E}{2} \times 2h\left[\frac{1 + 2c - 1}{1 + c^2}\right]$

$= c\,[mg + (Mg \pm f)] \times 2h \left(\frac{2c}{1+2c}\right)$

power $= \frac{4C}{1+2C}$ [mg + (Mg ± f)] ...

If friction neglected $f = 0$

Power $= \frac{4C^2}{1+2C}$ [mg + Mg] h

| power $= \frac{4C^2}{1+2C} (m + M) gr$ | by neglecting friction

[3]

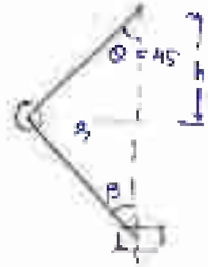
$K = 1$

$m = 1$

$M = 20$

$r = h$

$N = (?)$



$\cos 45° = \frac{h}{50}$

$\omega^2 = \frac{2mg + (Mg \pm f)(1+K)}{2mh}$ — h in 'm'

$= \frac{2mg + 2Mg}{2mh} = \left(\frac{m+M}{m}\right)\frac{g}{h}$

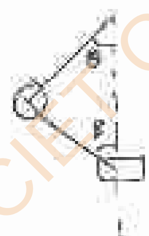
$= \left(\frac{1+20}{1}\right)\left(\frac{9.81}{0.35}\right)$

| ω = [illegible] rad/s |

[4]

$r = \sqrt{3}h$

$h = 20$ cm



$\omega^2 = \frac{2mg \pm (Mg \pm f)(1+K)}{2mh}$

$= \left(\frac{m+M}{m}\right)\frac{g}{h}$

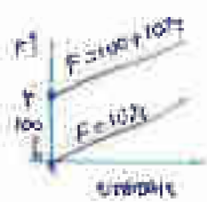
$= \left(\frac{2+10}{2}\right) \times \frac{9.81}{0.2}$

| ω = 17.15 rad/s |

[10]

$600 = P + q(50)$

$700 = P + q(60)$



| $q = 10$ | N/m $\quad 600 - 500 = P \Rightarrow$ | P = 100 | N

$\rightarrow F = 100 + 10r$

{ P < q else (ve) Governor unstable. to become isochronous P = 0, q > 0

CF = 50h − 1000 → 90V = 1

CF = 100h − 1000 → 80V = 3

(slope)$_c$ > (slope)$_2$

$x_3 > x_2 > x_1$

(i) ⇒ $\omega_1 > \omega_2 > \omega_3$

(ii) $x_1 = 40$ cm

$CF_1 = 0$

$CF_1 > 0$ ⇒ $x_2 > 40$ cm

⇒ $CF_3 > 0$ ⇒ $x_3 > 40$ cm

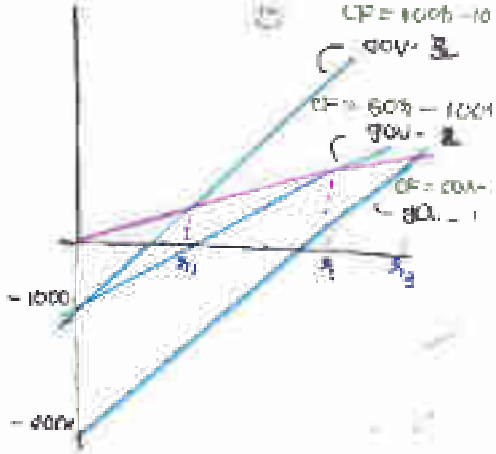
(iii) $60 < x < 60$

(e) $x_1 = 55$ ⇒ $CF_2 = 1400$

$CF_3 = 1400$

$(CF)_2 > (CF)_c$ → change in ... test / loading of governor / sleeve displacement test / concentric test } Governor ③

(iv) $\omega_1 < \omega_2 < \omega_3$



[b] CF ↓ — spring force ↓

↓ $F_s = kx$

$F_s \propto x$ ⇒ K ↓

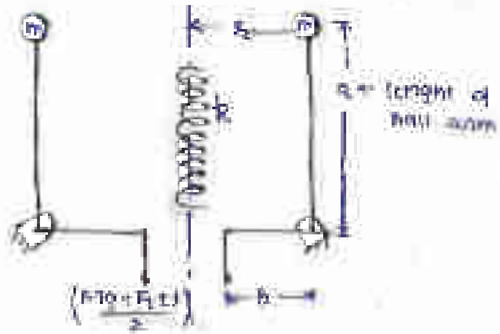
[7] 17 = p + 2q

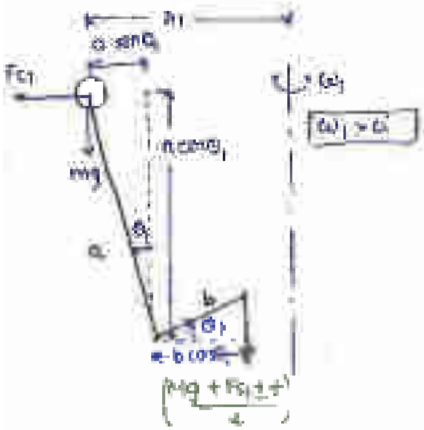
35 = p + 5q

18 = 4q ⇒ [q = 6] N/cm

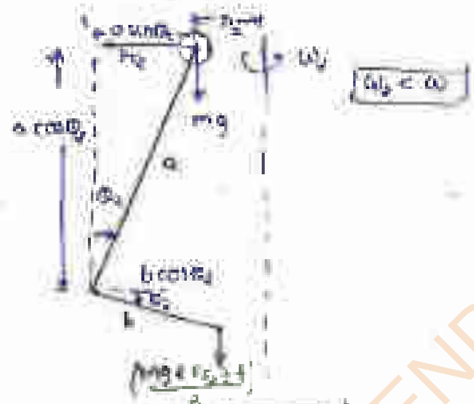
[p = 2] N

p > 0 ; q > 0 ⇒ unstable





case-(i) $\omega_1 > \omega$



case-(ii) $\omega_2 < \omega$

$$F_{c_1} \cdot a\cos\theta_1 + mg\, a\sin\theta_1 = \frac{Mg + F_{c_1} \pm f}{2} \cdot b\cos\theta_1$$

$$\Rightarrow m r_1 \omega_1^2\, a\cos\theta_1 + mg\, a\sin\theta = \frac{Mg + F_c \pm f}{2} \cdot b\cos\theta_1$$

$$F_{c_2}\, a\cos\theta_2 \pm mg\, a\sin\theta_2 = \frac{Mg + F_{c_2} \pm f}{2} \cdot b\cos\theta_2$$

$$\Rightarrow m r_2 \omega_2^2\, a\cos\theta_2 = mg\, a\sin\theta_2 + \left(\frac{Mg + F_c \pm f}{2}\right) b\cos\theta_2$$

→ Special case

If θ_1 & θ_2 are very small then

$$m r_1 \omega_1^2 \cdot a = \frac{Mg + F_{c_1} \pm f}{2} \cdot b$$

$$m r_2 \omega_2^2 \cdot a = \frac{Mg + F_{c_2} \pm f}{2} \cdot b$$

$$\Rightarrow \frac{r_1 \omega_1^2}{r_2 \omega_2^2} = \frac{Mg + F_{c_1} \pm f}{Mg + F_{c_2} \pm f}$$

If friction neglected

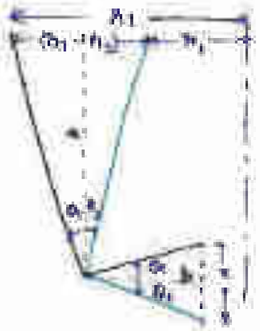
$$\frac{r_1 \omega_1^2}{r_2 \omega_2^2} = \frac{Mg + F_{c_1}}{Mg + F_{c_2}}$$

Condition for isochronous

$$\frac{r_1}{r_2} = \frac{Mg + F_{s_1}}{Mg + F_{s_2}}$$

Note: only spring controlled governors could be isochronous

→ Sleeve Displacement



$$\frac{x}{b} = \frac{(r_1 - r_2)}{a}$$

$$x = \frac{b}{a}(r_1 - r_2)$$

→ Stiffness of spring:

$$\left(\frac{Mg + F_{s_1} \pm f}{2}\right)b - \left(\frac{Mg + F_{s_2} \pm f}{2}\right)b = m r_1 \omega_1^2 a - m r_2 \omega_2^2 a$$

$$\Rightarrow \frac{b}{2}\left[Mg + F_{s_1} \pm f - Mg - F_{s_2} \pm f\right] = ma\left(r_1\omega_1^2 - r_2\omega_2^2\right)$$

$$\Rightarrow \frac{b}{2}\left[F_{s_1} - F_{s_2}\right] = \left(m r_1 \omega_1^2 - m r_2 \omega_2^2\right)a$$

$$F_{s_1} - F_{s_2} = (F_{c_1} - F_{c_2})\frac{2a}{b}$$

where $F_c = m r \omega^2$

$$\Rightarrow (F_s)_{net} = (F_{c_1} - F_{c_2})\frac{2a}{b}$$

$$K \cdot x = F_{c_1} - F_{c_2}\left(\frac{2a}{b}\right)$$

$$K = \frac{F_{c_1} - F_{c_2}}{\frac{b}{a}(r_1 - r_2)}\left(\frac{2a}{b}\right) \qquad \left\{ x = \frac{b}{a}(r_1 - r_2) \right.$$

$$K = 2\left(\frac{a}{b}\right)^2\left(\frac{F_{c_1} - F_{c_2}}{r_1 - r_2}\right)$$

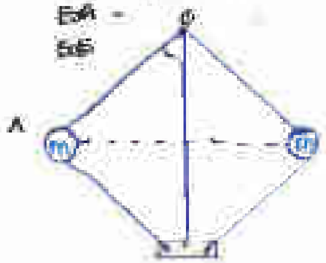
equilibrium speed so that calculate % change in speed for 50 mm rise in the level of balls.

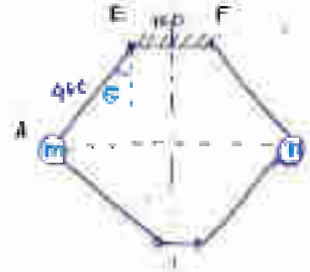
OA = 640 mm

θ = 30°

EA = 460

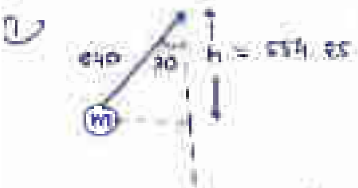
EF = 160, θ = 30





— simple watt governor

(1)



h = 0.554 m

$\omega_1^2 = \frac{g}{h_1} = \frac{9.81}{0.554}$

$\omega_1 = 4.20$ rad/s

(1)

h = 0.4156 m

$\tan 30 = \frac{80}{h'}$

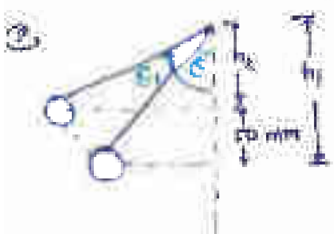
h' = 0.134 m

so, h_1 = 0.5591 m

$\omega_1^2 = \frac{g}{h} = \frac{9.81}{0.5591}$

ω_1 = 4.20 rad/s

(2)



h_2 = 0.554 − 0.05

h_2 = 0.504 m

$\omega_2^2 = \frac{g}{h} = \frac{9.81}{0.504}$

ω_2 = 4.4107 rad/s

% change speed = $\frac{N_2 - N_1}{N_1} \times 100$

= 4.64%

(2)

$\cos \theta_1 = \frac{0.3656}{0.460}$ ⇒ θ = 40.38°

$\tan 40.38° = \frac{80}{y}$ ⇒ y = 0.09426

total displa = 0.3656 + 0.09426

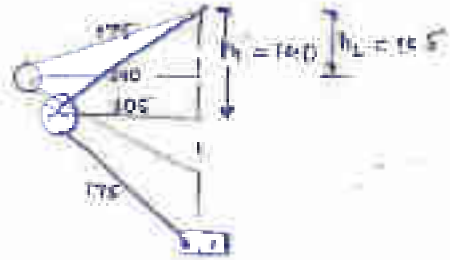
h_1 = 0.4546 m

$\omega_2 = \frac{g}{h_1} = \frac{9.81}{0.4476}$ ⇒ ω_2 = 4.619 rad/s

% $\frac{\omega_2 - \omega_1}{\omega_1}$ = 9.74%

The governor has radius of rotation at min & max speeds are 105 mm & 140 mm respectively. M = 20 kg, m = 5 kg. Determine range of speed
(i) when friction is absent
(ii) friction = 15 N.

→ arm length = 175 mm
$r_1 = 105$
$r_2 = 140$



Case-(i) Same length arm $k = 1$

→ $\omega_1^2 = \dfrac{2mg + (Mg \pm f)(1+k)}{2mh_1}$

$= \dfrac{(2)(5)(9.81) + [(20)(9.81) + 0](1+1)}{(2)(5)(0.140)}$

$\omega_1 = 15.71$ rad/s

Case-(ii) $\omega_2^2 = \dfrac{2mg + (Mg \pm f)(1+k)}{2mh_2}$

$= \dfrac{(2)(5)(9.81) + [(20)(9.81) + 0](2)}{(2)(5)(0.105)}$

$\omega_2 = 21.63$ rad/s

Range $= \omega_2 - \omega_1$

Range = 2.595 rad/s

Case-(ii) friction considered $k = 1$

take $h = \frac{r}{l}$ in $h_1' = 105$ & 140
$r = 5 \times 125$ to 105 ... more + f
... E (-ve)

→ $\omega^2 = \dfrac{mg + (Mg \pm f)}{mh}$

$\omega_1^2 = \dfrac{(5)(9.81) + (20)(9.81) - 15}{(5)(0.14)}$

r small F(-ve)
r big F(+ve)

$\omega_1 = 18.36$ rad/s

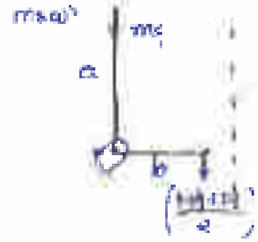
$\omega_2^2 = \dfrac{(5)(9.81) + (20)(9.81) + 15}{(5)(0.105)}$

$\omega_2 = 22.36$ rad/s

Range $= \omega_2 - \omega_1$

Range = 4.126 rad/s

Given $r_1 = 25$ cm
$\omega = 20$ rad/s
$K = 200$ N/cm
$a = b$



$$(m r_1 \omega^2)(a) = \left(\frac{F_s}{2}\right)(b)$$

$$(1)(0.25)(20)^2 = \frac{K x}{2} = \frac{(200)x}{2}$$

$$x = 1 \text{ cm}$$

6 $a = b = x/2$
$\omega = 20$
$r_2 = 40$ cm
$m = 1$ kg

$$(m r_2 \omega^2)\, a = \left(\frac{F_s}{2}\right)(b)$$

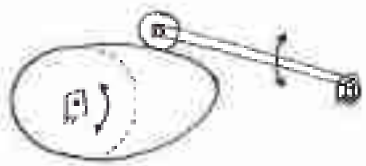
$$1 \times \frac{40}{100} \times (20)^2 \times \cancel{a} = \frac{F_s}{2}$$

$$F_s = 320 \text{ N}$$

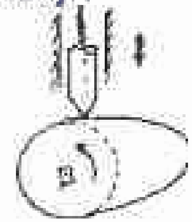
- It is Higher pair mechanism
- Cam is main rotating or oscillating element & follower follows the motion

→ Classification of cam & follower

1) On the basis of type of motion :
   a) Oscillatory cam & follower mechanism
   b) Translating cam & follower mechanism

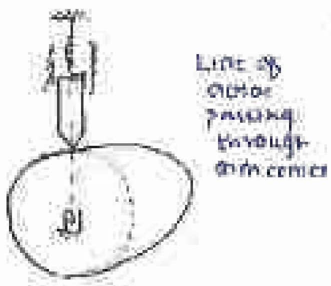


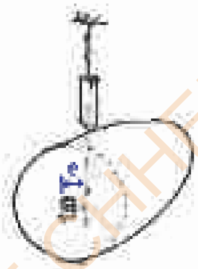
Oscillatory cam & follower



translating

2) On the basis of offset being provided
   a) Radial cam
   b) offset / Eccentric cam





3) On the basis of type of shape of follower
   a) Rise - Dwell - Return - Dwell [R-D-R-D]
   b) Dwell - Rise - Return - Dwell [D-R-R-D]

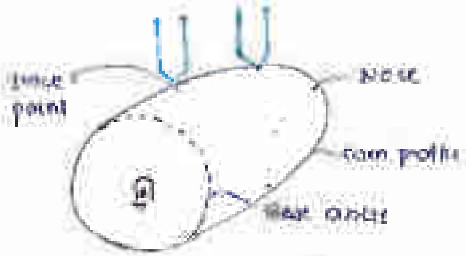
4) On the basis of the shape of follower
   a) knife edge follower 
   b) Roller follower 
   c) Flat face follower 
   d) Mushroom follower 

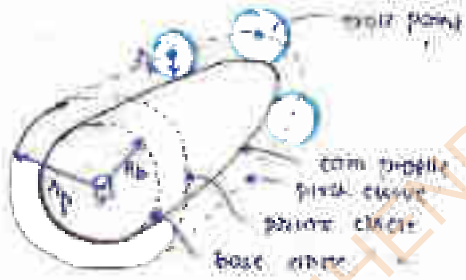
5) On the basis of type of motion of follower
   a) Uniform velocity
   b) uniform Acc^n or Retardation [Parabolic]
   c) Simple Harmonic [SHM]
   d) Cycloidal motion

- since, cam is main dimension rotating element but in order to discuss the terminology, we consider after inversion of cam & follower mechanism that is we consider cam as fixed member & follower is moving on it

① Base circle
- It is the smallest circle tangential to cam profile
- Radius of base circle determine the size of cam
- base circle will never intersect with cam profile

② Trace point :
- The reference point on follower whose locus is pitch curve is known as trace point
- In case of knife edge follower tip of the knife is trace point
- In case of roller follower center of roller is trace point

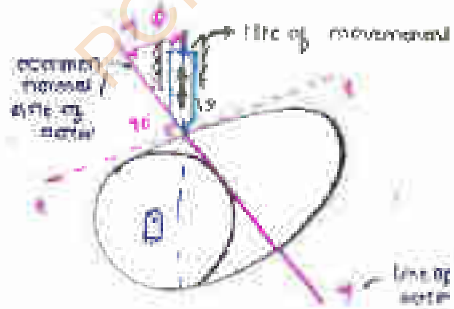




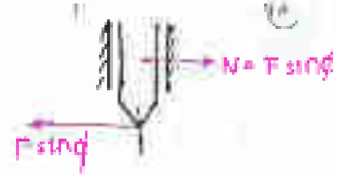
③ prime circle
- The smallest circle tangential to pitch curve is known prime circle

$R_{prime} = R_{base} + R_{roller}$

→ Pressure angle :
- It is the angle between direction of velocity of the follower (or line of movement of follower) and common normal at the point of contact
- pressure angle at for a cam profile is not constant
- In a cam & follower mechanism the force transmission always take place along the common normal that why it is also known as line of action



- A large value of pressure angle is avoided as it leads to Jamming.
- F sinφ component forms a couple for normal forces exerted by the guide and it tends to bend the stem of follower.



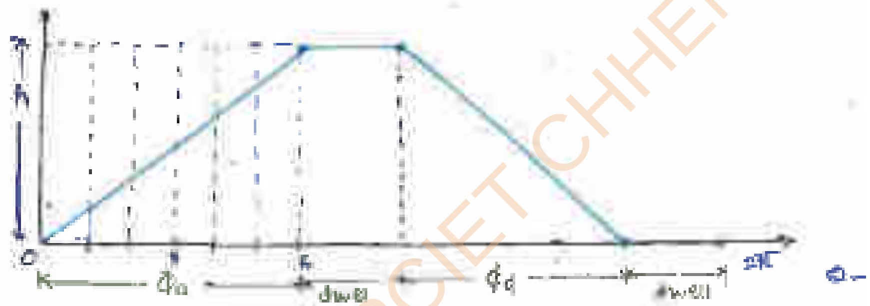
- The point on cam profile where pressure angle is maximum is known as pitch point.
- All the pitch point for cam profile results in a circle concentric to the base circle which is known as pitch circle.
- $\phi_{max} \leq 30°$

⇒ Types of motion of follower:

Note ① Displacement of follower is always measured from base circle.

Note ② In order to calculate pressure angle we used pitch curve (common normal drawn to pitch curve).

① Uniform velocity [R-D-R-D]



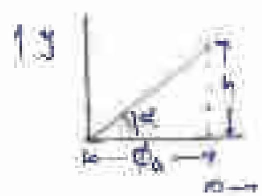
(Displacement diagram)

⇒ ϕ_a = angle of ascent / rise

ϕ_d = angle of descent / return

h = max distⁿ travelling by follower

→ Displacement eqⁿ



$y = const \cdot \theta$ (slope of line)

$\tan\alpha = \frac{h}{\phi_a}$

$$y = \left(\frac{h}{\phi_a}\right)\theta$$

$$v = \frac{dy}{dt} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{d}{d\theta}\left[\frac{h\theta}{\phi_a}\right]$$

$$\boxed{v = \frac{h\omega}{\phi_a}} = \text{const}$$

→ Accⁿ eqⁿ

$$a = \frac{dv}{dt} \qquad \text{Here } v = \text{const}$$

$$\boxed{a = 0}$$

→ Jerk equation

$$j = \frac{da}{dt}$$

$$\boxed{j = 0}$$

[2] Uniform acceleration or Retardation (parabolic motion)

$$y = a\theta^2 + b\theta + c \qquad @\ \theta = 0,\ y = 0$$
$$@\ \theta = \frac{\phi_a}{2},\ y = \frac{h}{2}$$
$$@\ \theta = 0,\ \frac{dy}{d\theta} = 0$$

1st condⁿ ⇒ $0 = a(0)^2 + b(0) + c$ ⇒ $\boxed{c = 0}$

2nd " ⇒ $\frac{dy}{d\theta} = 2a\theta + b$ ⇒ $\boxed{b = 0}$

3rd condⁿ ⇒ $y = a\theta^2$ ⇒ $\frac{h}{2} = a\left(\frac{\phi_a}{2}\right)^2$ ⇒ $\boxed{a = \frac{2h}{\phi_a^2}}$

→ Displacement eqⁿ

$$y = a\theta^2 \Rightarrow \boxed{y = 2h\left(\frac{\theta}{\phi_a}\right)^2}$$

→ Velocity eqⁿ

$$v = \frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt} = \omega\frac{dy}{d\theta} = \omega\frac{d}{d\theta}\left[2h\left(\frac{\theta}{\phi_a}\right)^2\right] = \frac{2h\omega}{\phi_a^2}\, 2\theta$$

$$\boxed{v = \frac{4h\omega}{\phi_a^2}\theta}$$

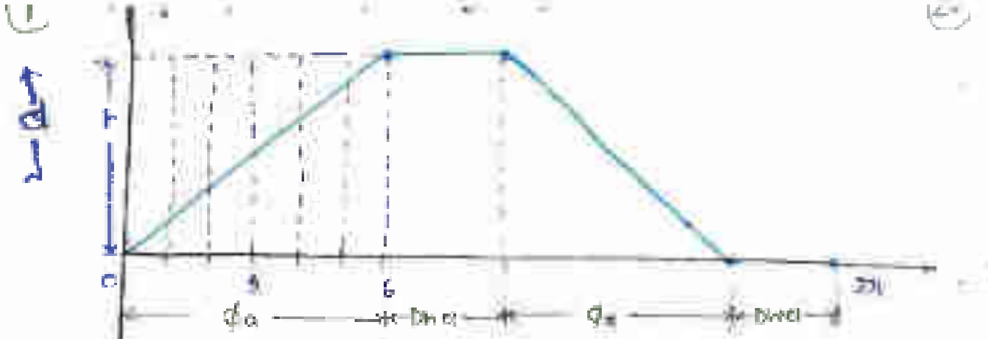
→ Accⁿ eqⁿ

$$a = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = \omega\frac{d}{d\theta}\left[\frac{4h\omega}{\phi_a^2}\theta\right]$$

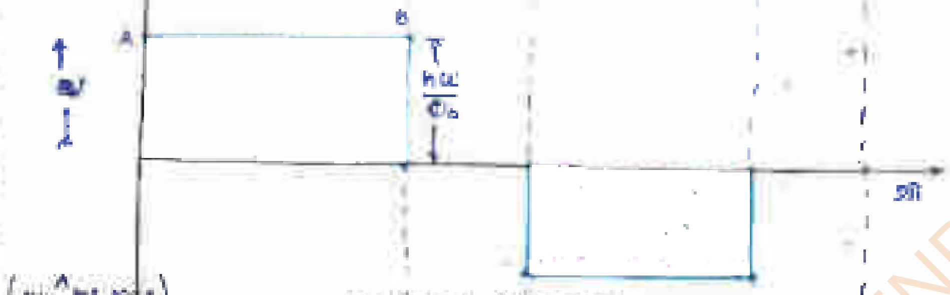
$$\boxed{a = \frac{4h\omega^2}{\phi_a^2}} = \text{const}$$

→ Jerk

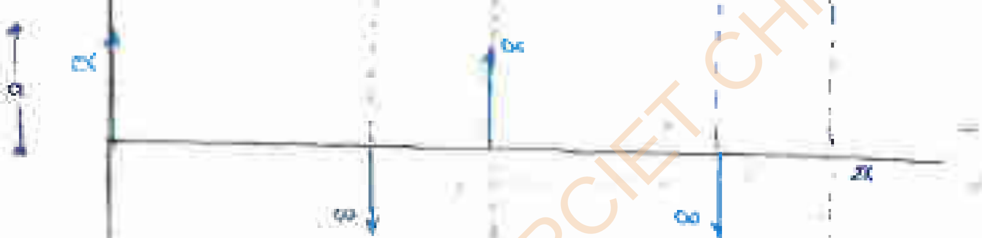
$$j = \frac{da}{dt} \Rightarrow \boxed{j = 0}$$



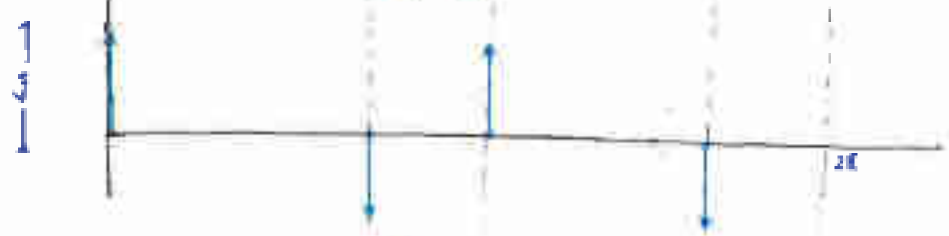
(i) Displacement diagram



(ii) velocity diagram

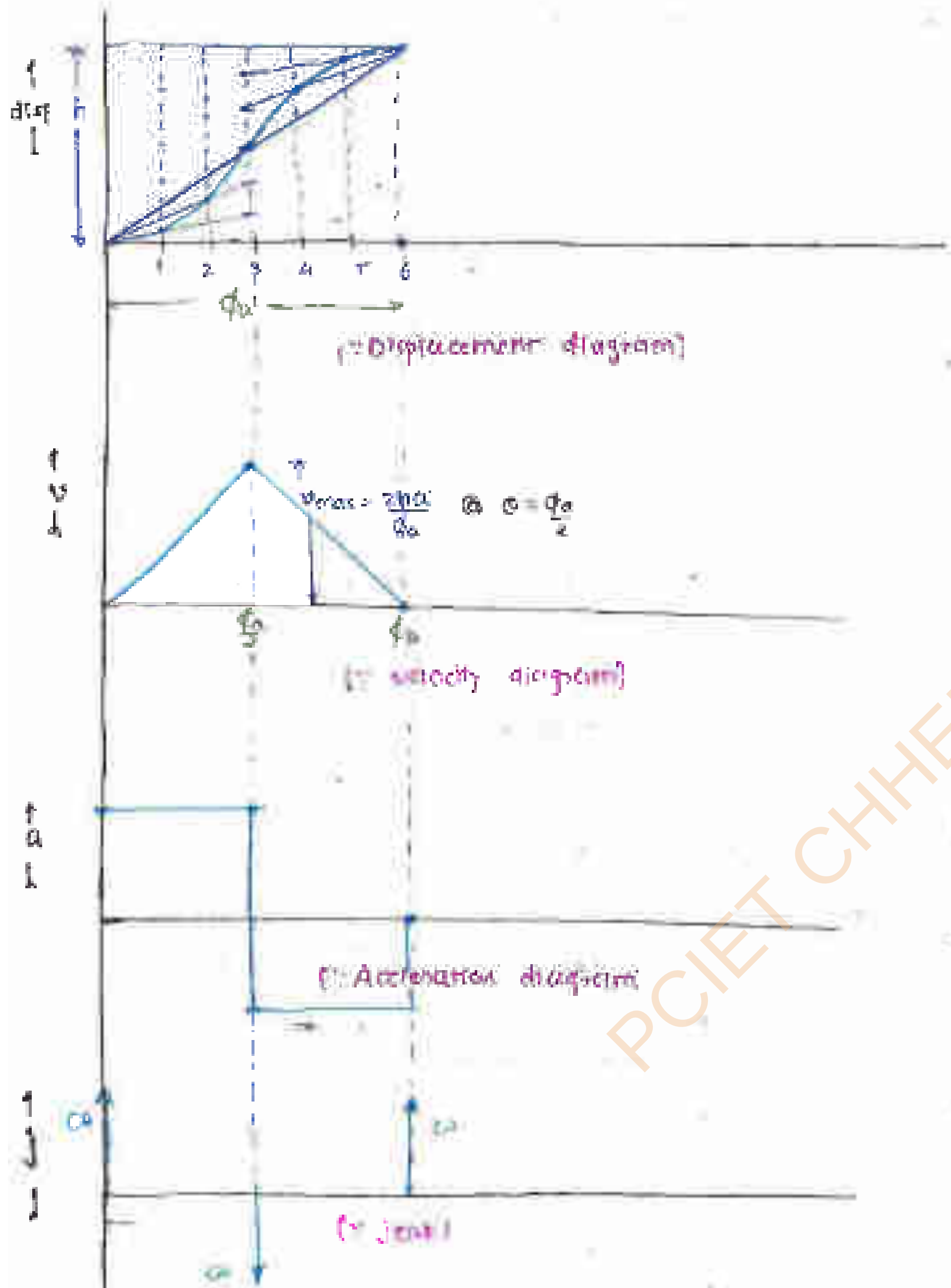


(iii) acceleration diagram

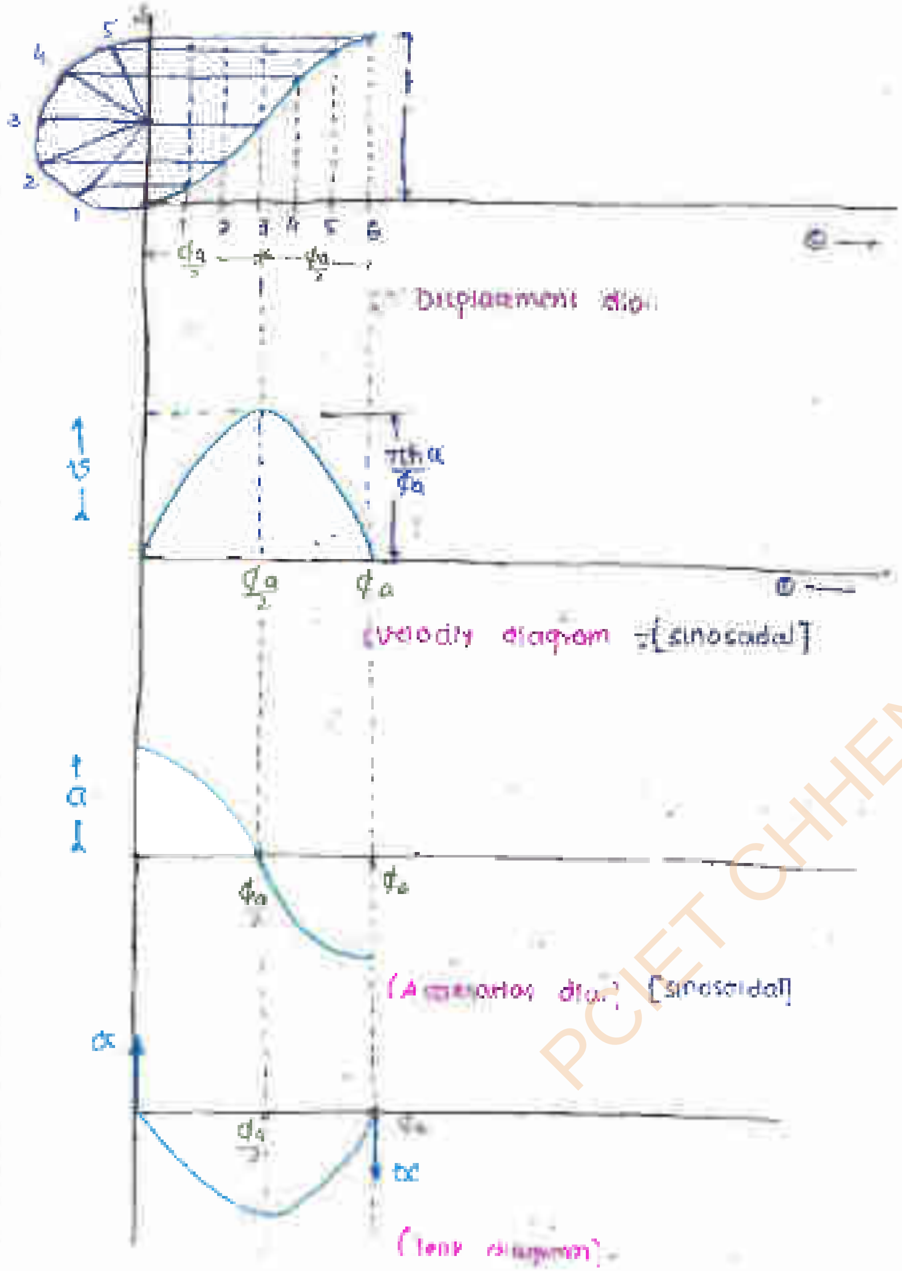


(iv) Jerk

[∵ uniform velocity φ-D-R-D]

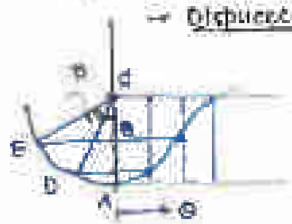


[∵ uniform accⁿ or retardation]



Displacement dia
θ
Velocity diagram :-[sinosoidal]
(Acceleration dia:) [sinosoidal]
(Jerk diagram)

→ Displacement eqⁿ



$AB = AC - BC = \frac{h}{2} - EC\cos\beta = \frac{h}{2} - \frac{h}{2}\cos\beta$

$y = \frac{h}{2}[1 - \cos\beta]$

$$\boxed{y = \frac{h}{2}\left[1 - \cos\left(\frac{\pi\theta}{\phi_a}\right)\right]}$$

at $\beta \to \pi$, $y \to h$, $\theta \to \phi_a$ $\Big\}$ $\frac{\beta}{\pi} = \frac{\theta}{\phi_a} = \frac{y}{h}$

→ velocity eqⁿ

$v = \frac{dy}{dt} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dt} = \omega\frac{dy}{d\theta} = \omega\frac{d}{d\theta}\left[\frac{h}{2}\left(1 - \cos\left(\frac{\pi\theta}{\phi_a}\right)\right)\right]$

$= \frac{h\omega}{2}\left[0 - \left(-\sin\left(\frac{\pi\theta}{\phi_a}\right)\cdot\frac{\pi}{\phi_a}\right)\right]$

$$\boxed{v = \frac{\pi h\omega}{2\phi_a}\sin\left(\frac{\pi\theta}{\phi_a}\right)}$$

θ	0	$\phi_a/2$	ϕ_a
v	0	$\frac{\pi h\omega}{2\phi_a}$	0

hence $\boxed{v_{max} = \frac{\pi h\omega}{2\phi_a}}$ @ $\theta = \frac{\phi_a}{2}$

→ Accⁿ eqⁿ

$a = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt} = \omega\frac{d}{d\theta}\left[\frac{\pi h\omega}{2\phi_a}\cdot\sin\left(\frac{\pi\theta}{\phi_a}\right)\right] = \frac{\pi h\omega^2}{2\phi_a}\cdot\cos\left(\frac{\pi\theta}{\phi_a}\right)\cdot\frac{\pi}{\phi_a}$

$$\boxed{a = \frac{\pi^2 h\omega^2}{2\phi_a^2}\cdot\cos\left(\frac{\pi\theta}{\phi_a}\right)}$$

θ	0	$\phi_a/2$	ϕ_a
a	$\frac{\pi^2 h\omega^2}{2\phi_a^2}$	0	$-\frac{\pi^2 h\omega^2}{2\phi_a^2}$

$$j = \frac{da}{dt} = \frac{d\theta}{dt}\cdot\frac{da}{d\theta} = \omega\,\frac{da}{d\theta}$$

$$= \omega\,\frac{d}{d\theta}\left[\frac{\pi^2\omega^2 h}{2\phi_a^2}\cdot\cos\left(\frac{\pi\theta}{\phi_a}\right)\right]$$

$$= \frac{\pi^2 h\omega^3}{2\phi_a^2}\left[-\sin\left(\frac{\pi\theta}{\phi_a}\right)\cdot\frac{\pi}{\phi_a}\right]$$

$$\boxed{j = -\frac{\pi^3 h\omega^3}{2\phi_a^3}\cdot\sin\left(\frac{\pi\theta}{\phi_a}\right)}$$

θ	0	$\frac{\phi_a}{2}$	ϕ_a
j	0	$-\frac{\pi^3 h\omega^3}{2\phi_a^2}$	0

[4] Cycloidal Motion

— Displacement eq^n

$$\boxed{y = \frac{h}{\pi}\left[\frac{\pi\theta}{\phi_a} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\phi_a}\right)\right]}$$

— Velocity eq^n

$$v = \frac{dy}{d\theta}\cdot\frac{d\theta}{dt} = \omega\,\frac{d}{d\theta}\left[\frac{h}{\pi}\left\{\frac{\pi\theta}{\phi_a} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\phi_a}\right)\right\}\right]$$

$$= \frac{h\omega}{\pi}\left[\frac{\pi}{\phi_a} - \frac{1}{2}\cos\left(\frac{2\pi\theta}{\phi_a}\right)\cdot\frac{2\pi}{\phi_a}\right]$$

$$\boxed{v = \frac{h\omega}{\phi_a}\left[1 - \cos\left(\frac{2\pi\theta}{\phi_a}\right)\right]}$$

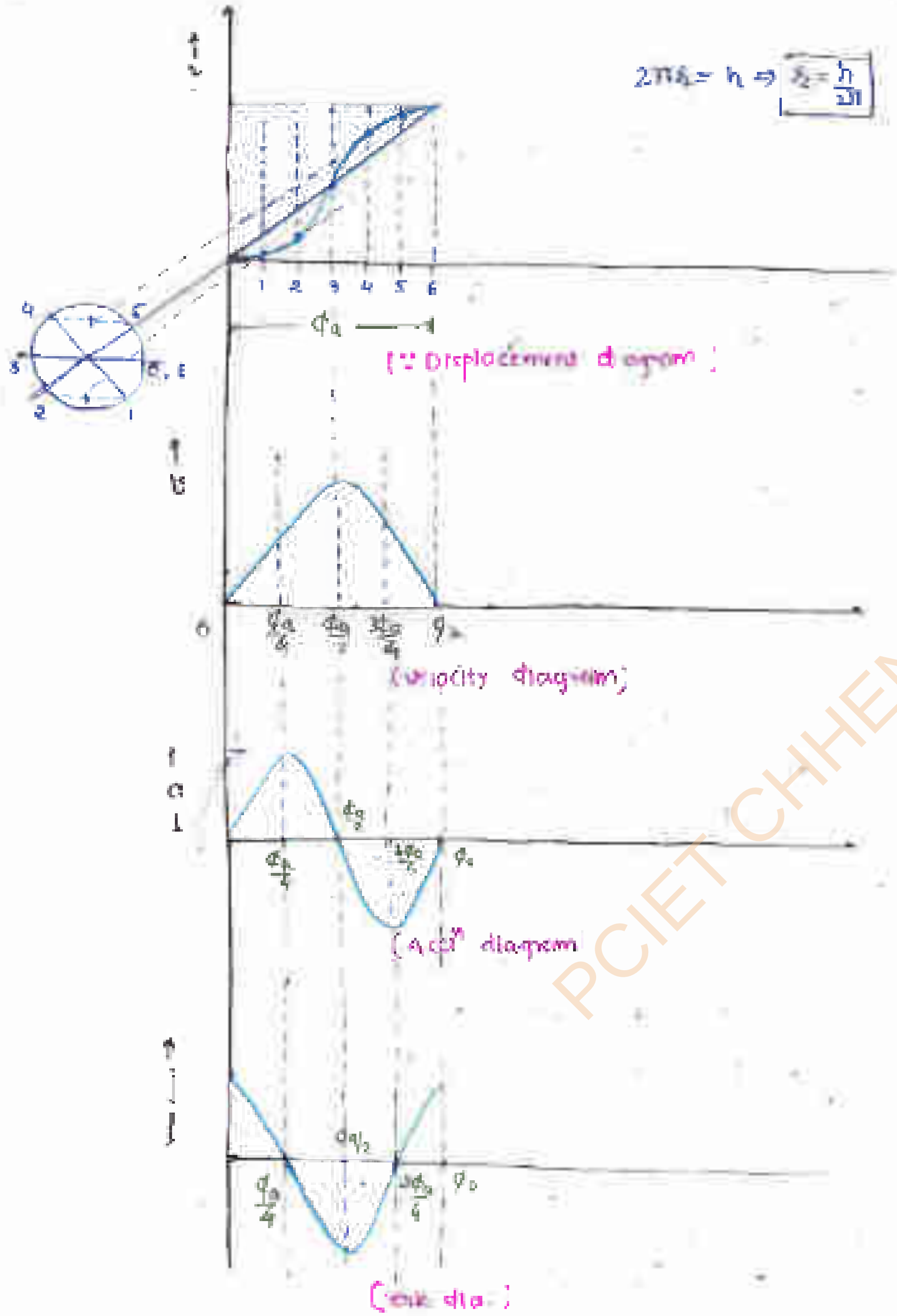
θ	0	$\phi_a/4$	$\phi_a/2$	$3\phi_a/4$	ϕ_a
v	0	$\frac{h\omega}{\phi_a}$	$\frac{2h\omega}{\phi_a}$	$\frac{h\omega}{\phi_a}$	0

— Acc^n eq^n

$$a = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt} = \omega\,\frac{d}{d\theta}\left[\frac{h\omega}{\phi_a}\left\{1 - \cos\left(\frac{2\pi\theta}{\phi_a}\right)\right\}\right]$$

$$= \frac{h\omega^2}{\phi_a}\left[0 - \left(-\sin\left(\frac{2\pi\theta}{\phi_a}\right)\right)\frac{2\pi}{\phi_a}\right]$$

$$\boxed{a = \frac{2\pi h\omega^2}{\phi_a^2}\sin\left(\frac{2\pi\theta}{\phi_a}\right)}$$



2πr₂ = h ⇒ r₂ = h/2π
1 2 3 4 5 6
(∵ Displacement diagram)
(velocity diagram)
(Acc^n diagram)
(Jerk dia.)

a	0	$\frac{2\pi h \omega^2}{\phi_a^2}$	0	$-\frac{2\pi h \omega^2}{\phi_a^2}$	0

$\rightarrow a_{max}$

→ jerk eq^n

$$j = \frac{da}{d\theta} \cdot \frac{d\theta}{dt} = \omega \frac{d}{d\theta}\left[\frac{2\pi h \omega^2}{\phi_a^2} \sin\left(\frac{2\pi\theta}{\phi_a}\right)\right]$$

$$j = \frac{2\pi h \omega^3}{\phi_a^2} \cos\left(\frac{2\pi\theta}{\phi_a}\right) \frac{2\pi}{\phi_a}$$

$$\boxed{j = \frac{4\pi^2 h \omega^3}{\phi_a^3} \cos\left(\frac{2\pi\theta}{\phi_a}\right)}$$

θ	0	$\phi_a/4$	$\phi_a/2$	$3\phi_a/4$	ϕ_a
j	$\frac{4\pi^2 h \omega^3}{\phi_a^3}$	0	$-\frac{4\pi^2 h \omega^3}{\phi_a^3}$		$\frac{4\pi^2 h \omega^3}{\phi_a^3}$

NOTE → cycloidal motion is best possible motion among all bove, hence it is used for high speed device
→ for moderate speed we can used SHM and for low speed uniform velocity

	V_{max}	a_{max}
uniform velocity	$\frac{h\omega}{\phi_a}$	—
SHM	$\frac{\pi h \omega}{2\phi_a}$	$\frac{\pi^2 h \omega^2}{2\phi_a^2}$
cycloidal	$\frac{2h\omega}{\phi_a}$	$\frac{2\pi h \omega^2}{\phi_a^2}$

$$\boxed{(V_{max})_{cycloidal} > (V_{max})_{SHM} > (V_{max})_{UV}}$$

[5]

$\phi_0 = 90°$

$\omega = 2$ rad/s

$\theta = \frac{2}{3}\phi_0$

$\rightarrow y = \frac{h}{2}\left[1 - \cos\frac{\pi\theta}{\phi_0}\right] = \frac{4}{2}\left[1 - \cos\pi\left(\frac{\frac{2}{3}\phi_0}{\phi_0}\right)\right]$

$\boxed{y = 3\ cm}$

$\rightarrow v = \frac{h}{2}\sin\left(\frac{\pi\theta}{\phi_0}\right)\cdot\frac{\pi}{\phi_0}\cdot\omega = \frac{\pi h\omega}{2\phi_0}\sin\left(\frac{\pi\theta}{\phi_0}\right)$

$= \frac{\pi \times 4 \times 2}{2 \times \pi/2}\sin(120)$

$\boxed{v = 7\ cm/s}$

$\rightarrow a = \frac{\pi h\omega^2}{2\phi_0}\cdot\cos\left(\frac{\pi\theta}{\phi_0}\right)\cdot\frac{\pi}{\phi_0} = \frac{\pi^2 h\omega^2}{2\phi_0^2}\cos\left(\frac{\pi\theta}{\phi_0}\right)$

$= \frac{\pi^2 \times 4 \times 2^2}{2 \times \pi^2/4}\cos(120)$

$\boxed{a = -16\ cm/s^2}$

[6]

$\phi_0 = \pi$

$h = 10$ cm

$v_{max} = 25$ cm/s

$\omega = (?)$

$a_{max} = (?)$

$v_{max} = \frac{\pi h\omega}{2\phi_0} = \frac{\pi \times 10 \times \omega}{2 \times \pi} = 25$

$\boxed{\omega = 5}$ rad/s

$a_{max} = \frac{\pi^2 h\omega^2}{2\phi_0^2} = \frac{\pi^2 \times 10 \times 5^2}{2 \times \pi^2} = \boxed{a_{max} = 125\ cm/s^2}$

[7]

$\phi_0 = \pi/2$

$h = 6$ mm

$\omega = 1$ rad/s

at $y = 3$ mm $\rightarrow$ $\boxed{\theta = \phi_0/2}$ (cuz at h 3 mm at half)

$v = \frac{\pi h\omega}{2\phi_0}\sin\left(\frac{\pi\theta}{\phi_0}\right) = \frac{\pi \times 6 \times 1}{2 \times \pi/2} \rightarrow \boxed{v = 6\ mm/s}$

$a = \frac{\pi^2 h\omega^2}{2\phi_0^2}\cos\left(\frac{\pi\theta}{\phi_0}\right) = \ldots\cos(\pi/2) \rightarrow \boxed{a = 0}$ mm/s²

$y = 10 + 5\sin\theta$

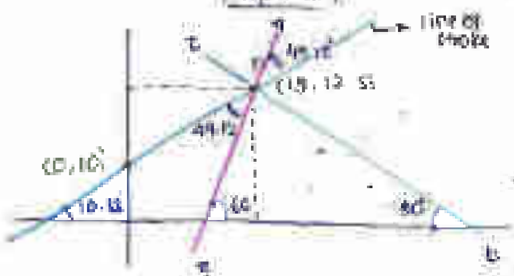
@ $\theta = 30$, $\boxed{\phi = ?}$

$\frac{x}{15} = \cos\theta$ & $\frac{y-10}{5} = \sin\theta$

$\Rightarrow \left(\frac{x}{15}\right)^2 + \left(\frac{y-10}{5}\right)^2 = 1 \Rightarrow C = (0, 10)$

at $\theta = 30$; $x_p = 15\cos 30$ $y_p = 10 + 5\sin 30$

$\boxed{x_p = 13}$ $\boxed{y_p = 12.5}$



$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \left(\frac{12.5 - 10}{13 - 0}\right)$

$\boxed{\theta_1 = 10.88}$

→ slope of tangent $= \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$= \frac{0 + 5\cos\theta}{-15\sin\theta}$

$= -\frac{1}{3\tan\theta}$

@ $\theta = 30°$ ⇒ $\frac{dy}{dx} = -\frac{1}{3\tan 30} = -\frac{1}{\sqrt{3}} = $

$\boxed{\alpha = -30°}$ from x-ax

→ $m_1 m_2 = -1$

$-\frac{1}{\sqrt{3}}\ m_2 = -1$

$m_2 = \sqrt{3}$

$\tan\beta = \sqrt{3}$

$\boxed{\beta = 60}$ from x-axis (ccw)

Angle bet^n com^n normal & line of stroke

$\boxed{Ans = 49.18°}$

[9] $y = 2x^2 - 7x - 2$ $z = 2(16) - 7(4) + c$ $= 32 - 28$

at $x = 4$, $y = 2$

→ Radial com line of action passing through centre

$\boxed{x_p = 4}$ & $\boxed{y_p = 2}$

$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{4 - 0}$

$\boxed{\theta = 26.56}$

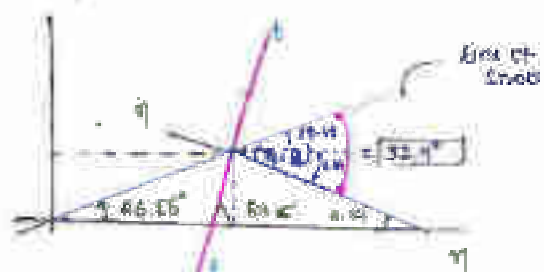
→ slope of tangent $\frac{dy}{dx} = \Big|_{(x_p, y_p) = (4, 2)}$

$= 4x - 7$

$= 16 - 7 = 9$

$\tan\beta = 9$

$\boxed{\beta = 83.65°}$



→ common normal

$m_1 m_2 = -1$

$m_2 = -1/9$

$\boxed{\alpha = -6.34°}$

(slope of tangent)(slope of normal) $= -1$

$\boxed{Ans = 32.9°}$

- it is angle betⁿ comⁿ normal to the pitch curve (or trace) point and line of stroke

— B_2 & B_3 are instantaneously coincident pt.

$V_{B_2} = OB\ \omega_2$

$\Rightarrow V_{B_3} = (I_{13}\text{-}B_3)\ \omega_3$

$V_{B_3} = (I_{23}\text{-}C)\ (\omega_2 = \omega_3)$

$$\boxed{V_{B_3} = (I_{23}\text{-}C)\ \omega_2}$$

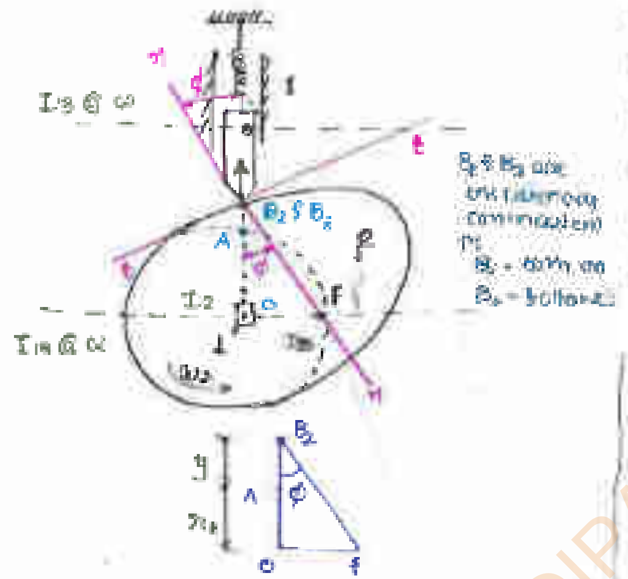
$V_{B_3} = (OP)\ \omega_2$

$\rightarrow \tan\phi = \frac{OP}{OB_3} = \frac{V_{B_3}/\omega_2}{OB_3}$

$\tan\phi = \frac{V_{B_3}}{\omega_2\ OB_3}$

$\rightarrow \tan\phi = \frac{dy/dt}{(R_b + y)\ d\theta/dt}$

$$\boxed{\tan\phi = \frac{dy/d\theta}{R_b + y}}$$



NOTE: - In case of Roller follower, $\tan\phi = \frac{dy/d\theta}{(R_b + R_R) + y}$

$$\boxed{\tan\phi = \frac{dy/d\theta}{(R_b + r_R) + y}}$$

where $R_b + R_R = r_{pitch}$

NOTE - In case offset provided

$$\boxed{\tan\phi = \frac{dy/d\theta - e}{\sqrt{r_b^2 - e^2} + y}}$$

when e - eccentricity being provided

NOTE:

- During rise stroke $\frac{dy}{d\theta}$ (+ve) therefore offset provided reduces pressure angle during rise.
- During return stroke $\frac{dy}{d\theta}$ (-ve) hence, offset increases pressure angle during return

cam center the cam should rotate A.C.W.
If it provided to left side of cam center cam should rotate C.W.

- A large value of ϕ should be avoided as it causes more friction, more wear, jamming, bending of the followers.

$N = F\sin\phi$ (in vg dir)
$f = \mu N$
$= \mu F\sin\phi$
$\rightarrow f = f^n(\sin\phi)$

As $\phi \uparrow \rightarrow$ friction $\uparrow$

- Larger value of ϕ_{max} are possible with parabolic & cycloidal motion
  & moderate value with SHM
  & smaller value for uniform velocity
  → (in order to above comparison)
  rise of followers,
  Angle of action (rise action) } same cond
  ω of cam

- The point of max velocity usually coincide with inflection point (The point where curvature is changing) at it the correspond to max. steepness of displacement of diagram

- parabolic & cycloidal motion required largest size of cam
  SHM → moderate size
  uniform velocity → smallest size

[*] Important point for selection of cam profile
1) there should be smooth jerkless motion
2) size of cam should be smaller
3) inertia of cam should not be much large
4)

[*] Important point for followers
1) knife edge follower
   ↳ simplest follower
   ↳ contact stresses are large
   ↳ result in more wear & tear
   ↳ knife-edge follower can be called as roller follower have zero radius

[4]

↳ the sliding friction of knife-edge follower is converted into rolling friction

↳ If the cam profile is steep roller follower gets jam

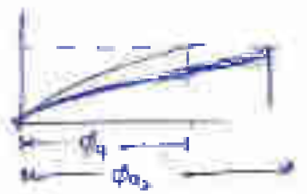
↳ It ensures high quality

[5] Flat face follower

↳ It can be used for relatively steep cam

↳ In order to minimize the contact stress we provide a spherical ends at the face and follower known as mushroom follower.

[1] $\tan\phi = \frac{dy/d\theta}{R_b + y}$



$v = \frac{dy}{dt} = \frac{dy}{d\theta} \times \frac{d\theta}{dt}$

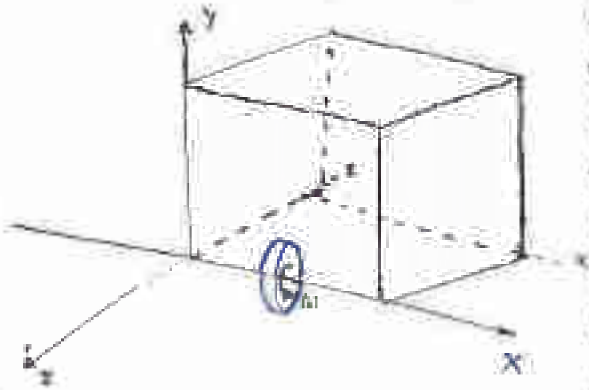
→ Motion	Displacement eqn	velocity	accⁿ	Jerk
1) Const uniform velocity	$y = \left(\frac{h}{\phi_a}\right) \cdot \theta$	✓	0	0
2) uniform accⁿ	$y = 2h\left(\frac{\theta}{\phi_a}\right)^2$	✓ V_{max} @ $\theta = \frac{\phi_a}{2}$	✓	0
3) SHM	$y = \frac{h}{2}\left[1 - \cos\left(\frac{\pi\theta}{\phi_a}\right)\right]$	✓ V_{max} @ $\theta = \frac{\phi_a}{2}$	✓ a_{max} @ $\theta = 0$ & ϕ_a	✓
4) cycloidal	$y = \frac{h}{\pi}\left[\frac{\pi\theta}{\phi_a} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\phi_a}\right)\right]$	✓ V_{max} @ $\theta = \frac{\phi_a}{2}$	✓ a_{max} @ $\theta = \frac{\phi_a}{4}$ & $\frac{3\phi_a}{4}$	✓ J_{max} @ $\theta = 0$ & ϕ_a & $\frac{\phi_a}{2}$

→ Cylindrical cam have reciprocating motion of follower.

→ Roller follower used in engine.

Spin motion

- The rotation of motor (or engine parts) is known as spin motion.
- The the plane on which it can be observed or parallel to it is called a plane of spin.



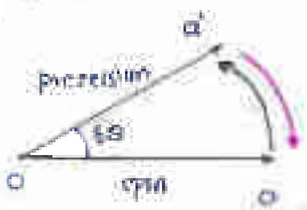
Precession

- The rotation or oscillating of axis of spin (that is main rotating element) is called precession.

→ Whenever the main rotating object starts to process about same axis it result in change in its angular momentum which gives rise to couple known as gyroscopic couple.

→ This gyroscopic couple tends to change the position of the object.

→ There will be an reactive gyroscopic couple exerted by the bearing whose magnitude equals to active gyroscopic couple but direction will be opposite & it will always give the same effect.



- Observer at 'o'
- Rotor is rotating 'cw'
- shaft is moving towards left

	Axis	plane
spin	+x	yz
precession	+y	xz
Reactive	+z	xy

$aa' = oa\,(\delta\theta)$

$\frac{d}{dt}(aa') = \frac{d}{dt}(oa \cdot \delta\theta)$

$\frac{d}{dt}(aa') = oa\,\frac{d}{dt}(\delta\theta) \Rightarrow \boxed{\frac{d}{dt}(aa') = (oa)\,\omega_p}$

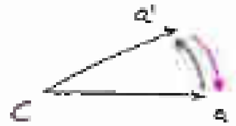
$\left\{ \begin{array}{l} F = \frac{d}{dt}(mv) \\ C = \frac{d}{dt}(I\omega) \end{array} \right.$

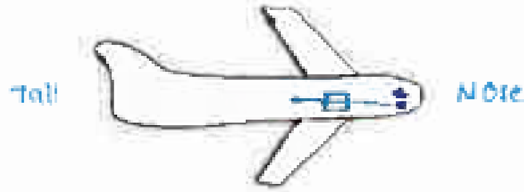
$\boxed{C = I\omega_s\omega_p}$ N-m

→ Rotate 90° the vector of RH in spin dir & then curl the fingers in dirn of moving then thumb indicate the gyroscopic dirn. (vector of RH in spin dirn)

① On aeroplane

- Observer is on tail side
- Rotor rotating cw.
- Plane turning





plane will moves towards right

	Axis	plane
spin	+x	yz
precession	+z	xy
Rec' and	−y	xz

→ Obs: plane is taking right turn

Ex Observer tail side.
Rotating rotor c.w.
plane taking right turn

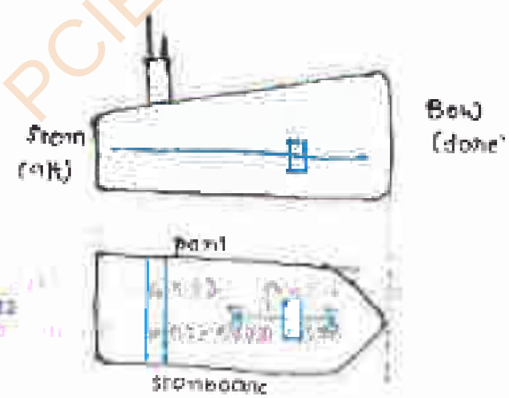
	Axis	plane
spin	+x	yz
precession	−y	xz
Rec' and	−z	xy

Nose will come down.

② Naval ships

Steering

- If ship is taking right or left turn on curved path



Pitching

- oscillation of ship about transverse axis

Rolling

- oscillation of ship about longitudinal axis

- The axis which passing through c/s is longitudinal / plane axis from other axis transverse axis.

- Observer is standing in stern side
- Ship is moving towards port side
- Rotor is rotating c/w

	Axis	Plane
Spin	x	yz
Precession	y	xz
Reactive gyro.	z	xy

→ Bow will raise

2. Effect of Gyroscopic couple in pitching

Observer is standing stern side
Bow is coming down
Rotor is rotating c/w

	Axis	Plane
Spin	x	yz
Precession	−z	xy
Rea. gyro	y	xz

→ towards port

Effect of Gyroscopic couple in Rolling

Observer is standing stern side
Rotor is rotating c/w
Ship is Rolling

	Axis	Plane
Spin	+x	yz
Precession	+x/−x	yz
Rea. gyro	−	−

In rolling there is no gyroscopic couple.
(as the axis of precession of is parallel to ... small effect of ... of is ...)

- In steering :

$$C = I\omega_s\omega_p$$

where ω_s = angular speed of spin
ω_p = " " of precession (where vehicle is moving turn)
I = mass MOI of rotor

$$\omega_p = \frac{v}{R}$$

- In pitching :
  - Assume it is SHM.

$$\theta = \theta_0 \sin \omega t$$

where $\theta_{max} = \theta_0$ → precession
= max angular displacement from the mean position

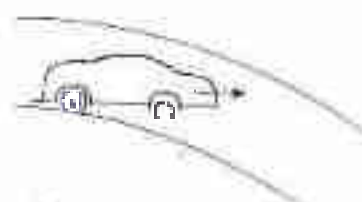
Displacement eqⁿ $\theta = \theta_0 \sin \omega t$ → $\theta_{max} = \theta_0$
velocity eqⁿ (precession) $\dot{\theta} = \theta_0 \omega \cos \omega t$ → $(\omega_p)_{max} = \dot{\theta}_{max} = \theta_0 \omega_1$
Accⁿ eqⁿ $\ddot{\theta} = -\theta_0 \omega^2 \sin \omega t$ → $(\ddot{\theta})_{max} = \theta_0 \omega^2$

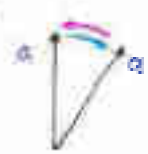
$\ddot{\theta} \propto -\omega^2 \theta$
accⁿ ∝ − Displacement

→ Acceleration ∝ (−Displacement) to show SHM motion
that displacement is a force or accⁿ acting is opposite dirⁿ

➡ Effect of Gyroscope in Automobile :
➡ Vehicle is moving in forward direction &
going to turn a right turn
Engine rotating similar to wheels

	Axis	Plane
Spin	+x	yz
Precession	−y	xz
Reactive axis	+z	xy





	Axis	Plane
Spin	−z	xy
Precession	−y	xz
Reac. gy.	−x	yz

iii) $R = 100$ m

$v = 20$ m/s

$\omega_s = 100$ rad/s

$I = 10$ kg m²

$C = I\,\omega_s\,\omega_p$

$= I\,\omega_s \left(\frac{V}{R}\right)$

$= 10 \times 100 \times \left(\frac{20}{100}\right)$

$\boxed{C = 200 \text{ N·m}}$

[9] $m = 6000$ kg

$N = 2400$ rpm $\rightarrow$ $\omega_s = 251.2$ rad/s

dirⁿ of rotation of rotor is c.w. viewed from stern

$R = r_g = 450$ mm

$I = mk^2 = 6000\,(0.45)^2$

$= 1215$ kg m²

(i) Steering (steers to left) (port side)

$R = 60$ m

$V = 16$ knot $= 1860 \times 16$ m/hr $= 9.5$ m/s

$\omega_p = \frac{9.5}{60} = 0.155$ rad/s

The $C = I\,\omega\,\omega_p$

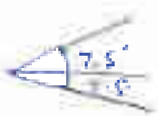
	Axis	Plane
Spin	+X	YZ
Precession	+Y	XZ
Reacⁿ gy	Z	XY

Bow up (rise)

$C = (1215)(0.155)(251.2)$

$\boxed{C = 47.3 \text{ kN·m}}$

ii) Pitching



Bow is descending

$\boxed{\theta = \theta_0 \sin \omega t}$ (pitching is SHM)

$\theta_{max} = \theta_0 = 7.5 \times \frac{\pi}{180} = 0.13089$

$\dot{\theta} = \theta_0\,\omega \cos \omega t$

$\dot{\theta}_{max} = \theta_0\,\omega$; $\omega = \frac{2\pi}{T} = \frac{2\pi}{18} = 0.3490$

$= (0.13089)(0.3490)$

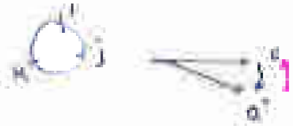
[illegible]

$= (1215)(251.2)(0.04561)$

$\boxed{C = 13.1 \text{ kN m}}$

	Axis	plane
spin	+x	yz
precession	+z	xy
Reaⁿ gyro	+y	xz

(descending)



Ship is raising left turn → on port side ↑ (ccw)

$\dot{\omega}_{max} = \phi_0 \omega_1^2 = \underline{0.0156}\ rad/s^2$

(iii) $\omega_p = 0.035\ rad/s$

$\omega = 80\pi\ rad/s$

$C = I\omega\omega_p = 1215(80\pi)(0.035)$

$\boxed{C = 10.68 \text{ kN m}}$

No effect of gyroscopic coupling ↗

(c) $N = 3600$ rpm → $\omega_s = 314.0\ rad/s$

$I = 47.25\ kg\ m^2$

$\omega_p = \frac{2\pi}{T}$; $T = 17$ sec

$\omega_p = \frac{2\pi}{17} = 0.3694$

$C = I\omega_s\omega_p = (47.25)(314)(0.3594)$

$\boxed{C = 5.48 \text{ kN-m}}$

	Axis	plane
spin	+x	yz
precession	−y	xz
Reaⁿ gyro	−z	xy

Bow will come down ↓

4

	Axis	Plane
spin	+x	yz
prec	+y	xz
Reac^n arm	+z	xy

Nose raise.

2

	Axis	plane
spin	+x	
prec	-y	
Rea^n a	-z	

depress bow

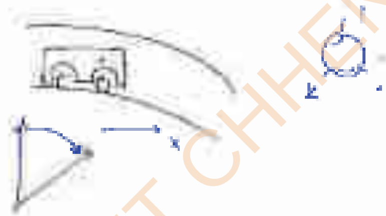
3

+x
+z
-y

turn ship towards star board

5

+z
-y
+x



~~outer wheel~~
inner wheel

6

+x   yz
+z/-z   xy
+y/-y   xz

in Horizontal plan.

Ex A uniform disc of 200 mm diameter has mass of 10 kg it is mounted of centralize on the horizontal shaft which rung in bearing, which are 150 mm apart. At spin = 2000 rpm in c.c.w dirn rotating from right hand side bearing shaft precessing uniform velocity of 60 rpm in Horizontal plane in A.C.W. when looked from top determine reaction at each bearing due to mass & gyroscopic effect

→ R = 100 mm
m = 10 kg

$\omega_s = \frac{2\pi N_s}{60} = \frac{2\pi(2000)}{60} = 209.33$

$\omega_p = \frac{2\pi N_p}{60} = \frac{2\pi(50)}{60} = 5.23$

Ring	$I = mR^2$
Disc	$I = \frac{mR^2}{2}$

$C = I\omega_s\omega_p \Rightarrow$ $\boxed{C = 54.60 \text{ N-m}}$

$\rightarrow P_{A_1} = P_{B_1} = \frac{Mg}{2} = \frac{(10)(9.81)}{2} \Rightarrow \boxed{P_{A_1} = P_{B_1} = 49.05}$

$\rightarrow P_{A_2}$ & P_{B_2} is reaction due to gyro couple

$(P_A)_2 \times \ell = 54.60 \Rightarrow (P_A)_2 = \frac{54.60}{0.15}$

$\boxed{(P_A)_2 = 356.2 \text{ N}} = (P_B)_2$

$(P_B)_{net} = \sqrt{\quad}$

Axes	Axis	Plane
Spin	+x	yz
precn	+y	xz
Rea' couple	+z	xy

$(P_A)_{net} = 49.05 - 365.54 = -316.4$

$(P_B)_{net} = 49.05 + 365.54 = 414.57$

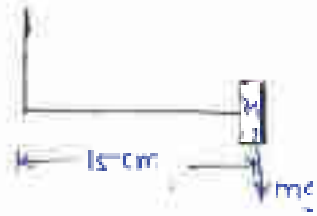
It is given spin of 1000 rpm about its axis which horizontal flywheel is suspended at a point 15 cm from the plane of rotation of flywheel determine the motion of the flywheel (ωp) = (?)

$m = 10$ kg

$k = 0.2$ m

$I = \frac{mk^2}{2} \Rightarrow \boxed{I = 0.4}$ kg m²

$N_{spin} = 1000$ rpm



$Mg \cdot l = I\omega_s \omega_p$

$10 \times 9.81 \times \frac{15}{100} = 0.4 \times \left(\frac{2\pi \times 1000}{60}\right) \omega_p$

$\boxed{\omega_p = 0.352 \text{ rad/s}}$

Ex A thin disc of mass m & Radius R is mounted on a light rod of length 2a which is freely hinged at O & other end of rod, & other end of rod is being supported by a light string. disc spin with an angular speed ω as shown in figure & whole assembly rotate about a vertical axis through O with an angular ωp. determine the tension in spring.

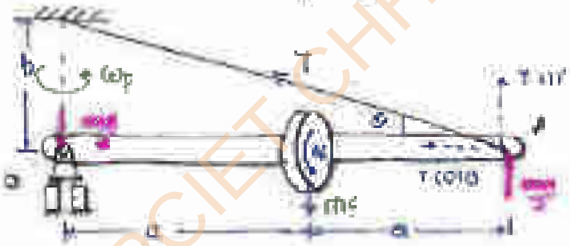
→ $C = I\omega\omega_p$

$\Sigma M_x = 0$

$mg(2a) + mg(a)$

Acw ctrl

$T \sin\theta (2a) - mg(a) + I\omega\omega_p = C$



	Axis	Plan
spin	+x	YZ
prece	+Y	XZ
Prec. ax.	+Z	YZ

$T \sin\theta \, 2a = mg\,a - \left(\frac{mR^2}{2}\right)\omega\omega_p$

$T = \left\{mga - \left(\frac{mR^2}{2}\right)\omega\omega_p\right\} \frac{1}{2a \sin\theta}$

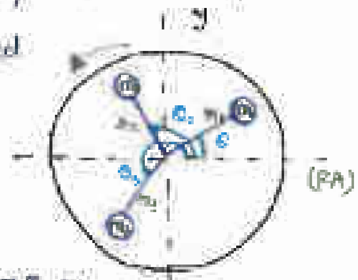
but $\sin\theta = \frac{h}{\sqrt{a^2+h^2}}$

$T = \left\{mga - \left(\frac{mR^2}{2}\right)\omega\omega_p\right\} \frac{\sqrt{4a^2+h^2}}{2a\,h}$

- process of removing unbalanced force or unbalanced couple either by adding some extra mass or by removing excess mass known as balancing

→ **Types of Balancing :**

(i) **Static Balancing :**
- when the masses are rotating in same plane



→ If system is statically balanced

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$m_1 r_1 \omega^2 \cos\theta_1 + m_2 r_2 \omega^2 \cos\theta_2 + m_3 r_3 \omega^2 \cos\theta_3 = 0$

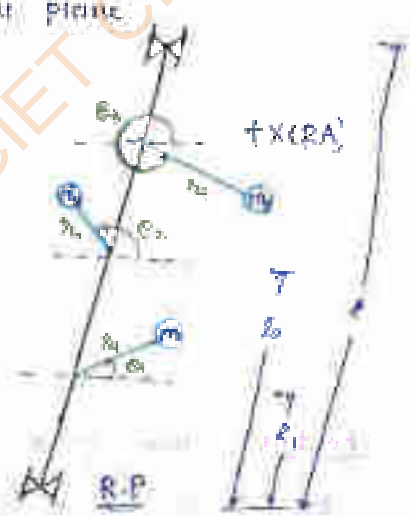
$m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 = 0$

$\boxed{\Sigma m_i r_i \cos\theta_i = 0}$ & $\boxed{\Sigma m_i r_i \sin\theta_i = 0}$

→ If a system is statically balanced then force polygon will be closed

→ In statically balanced system the reaction will be equal in magnitude & same in direction

(ii) **Dynamic Balancing :**
- when the masses lies on different plane
- If the system is dynamically balanced then force polygon as well as couple polygon will be closed



→ $\Sigma F_x = 0$

$\boxed{\sum_{i=1}^{n} m_i r_i \cos\theta_i = 0}$

→ $\Sigma F_y = 0$

$\boxed{\sum_{i=1}^{n} m_i r_i \sin\theta_i = 0}$

→ $m_1 r_1 \omega^2 l_1 \cos\theta_1 + m_2 r_2 \omega^2 l_2 \cos\theta_2 + \dots = 0$

$m_1 r_1 \omega^2 l_1 \sin\theta_1 + m_2 r_2 \omega^2 l_2 \cos\theta_2 + \dots = 0$

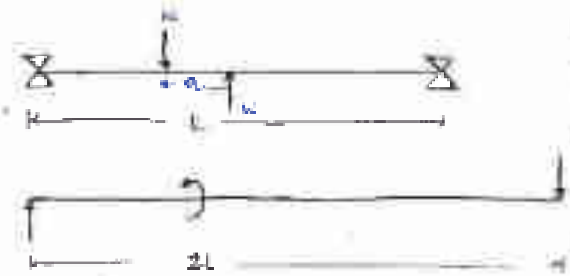
$\boxed{\sum_{i=1}^{n} m_i r_i l_i \cos\theta_i = 0}$ and $\boxed{\sum_{i=1}^{n} m_i r_i l_i \sin\theta_i = 0}$

be opposite in direction.

(iii) complete balancing

- If the system is statically as well as dynamically balanced it is said to be completely balance.
- Reaction will be different in magnitude as well as direction.

CRQ
-[1]



case

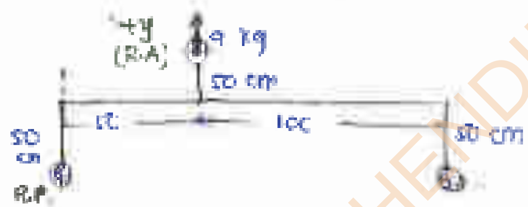
$R \cdot L = W \cdot a$

$$\boxed{R = \frac{Wa}{L}}$$

$\rightarrow R' = \frac{Wa}{2L}$

$$\boxed{R' = R/2}$$

[2]



mass	r	θ	mr cosθ	mr sinθ	l	mrl cosθ	mrl sinθ
B_1	50	180	$B_1(50)\cos 180$	$B_1(50)\sin 180$	0	0	0
9	50	0	$9(50)\cos 0$	$9(50)\sin 0$	50	$9(50)(50)$	0
B_2	50	180	$B_2(50)\cos 180$	$B_2(50)\sin 180$	150	$B_2(50)\cos 180(150)$	0

$\rightarrow \Sigma mrl\cos\theta = 0 \Rightarrow 9(50)(50) = B_2(50)(150)$

$$\boxed{B_2 = 3\ kg}$$

$\rightarrow \Sigma mr\cos\theta = 0 \Rightarrow -B_1(50) + 9(50) - B_2(50) = 0$

$-150 + 450 = B_1(50)$

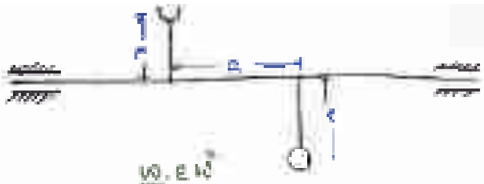
$$\boxed{B_2 = 6\ kg}$$

⇒ Shortcut :- when system is completely balanced then ..

$B_1 = \frac{9 \times 100}{150}$ & $B_2 = \frac{9 \times 50}{150}$

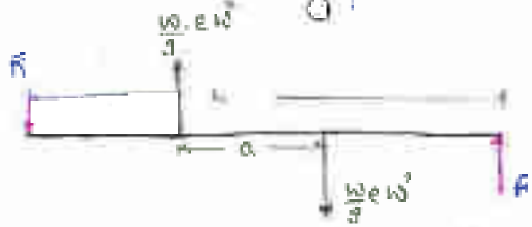
$\boxed{B_1 = 6\ kg}$ $\boxed{B_2 = 3\ kg}$

(4) Dynamic reaction are opposite in nature.



$R \cdot L = m \frac{W}{g} e\, \omega^2 \cdot a$

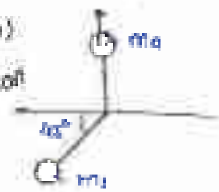
$$R = \frac{W}{g} e\, \omega^2 \frac{a}{L}$$



[6] & [7]

$m_A \to R.P$

m_c & $m_D \to R.P$ (become equal) (entity both) entity out in the ...



mass	r	θ	mr cosθ	mr sinθ	l	mrl cosθ	mrl sinθ
5	20	0	(5)(20)	0	−20	−(5)(20)(20)	0
(R.P) m_c	20	θ_c	$m_c(20)\cos\theta_c$	$m_c(20)\sin\theta_c$	0	0	0
m_D	20	θ_D	$m_D(20)\cos\theta_D$	$m_D(20)\sin\theta_D$	40	$m_D(20)\cos\theta_D$ (40)	$m_D(20)\sin\theta_D$
6	20	135°	(6)(20) cos135°	(6)(20) sin135°	60	(6)(20) cos135° (60)	(6)(20) sin135°

$\Rightarrow \Sigma mrl \cos\theta = 0 \quad \Sigma mrl \sin\theta = 0$

$0 + 0 + m_D(20)(\sin\theta_D)(40) + (6)(20)\sin 135°(60) = 0$

$800\, m_D \sin\theta_D = -5091.1$

$$m_D \sin\theta_D = -6.364 \quad \text{--- (I)}$$

$\Sigma mrl \cos\theta = 0$

$-(5)(20)(20) + m_D(20)\cos\theta_D(40) + (6)(20)(\cos 135°)(60) = 0$

$-2000 + 800\, m_D \cos\theta_D - 5091.168 = 0$

$$m_D \cos\theta_D = 8.864 \quad \text{--- (II)}$$

$\tan\theta_D = -\frac{6.364}{8.863}$

$$\theta_D = -35.69 \quad \text{from} \quad \theta_D = 324.30°$$

$m_D^2 \cos^2\theta_D + m_D^2 \sin^2\theta_D = (6.364)^2 + (8.863)^2$

$$m_D = 10.419 \text{ kg}$$

→ $20\, m_c \sin\theta_c + (20)(10.41)\sin(-35.63) + (8)(20)(0.707) = 0$

$m_c \sin\theta_c = 4.119$ — (iii)

$\Sigma\, m r \cos\theta = 0$

→ $20\, m_c \cos\theta_c + (20)(10.41)\cos(-35.63) - (8)(10)(0.707) = 0$

$m_c \cos\theta_c = -9.621$ — (iv)

$m_c^2 = (-2.119)^2 + (-4.621)^2$ $m_c^2 = (+2.119)^2 + (-9.621)^2$

$m_c = 5.08$ kg $m_c = 9.85$ kg

$\tan\theta_c = \frac{-2.119}{-4.621}$ $\tan\theta_c = \frac{+2.119}{-9.621}$

$\theta_c = 24.63$ $\theta_c = -12.45$, 167.5

167.5°

→ time period for tan is 180° or repeat after a 180°

NOTE →

$\sin\theta \rightarrow 2\pi$ $\cos\theta \rightarrow 2\pi$ $\tan\theta \rightarrow \pi$

$\sin n\theta \rightarrow \frac{2\pi}{n}$ $\cos n\theta \rightarrow \frac{2\pi}{n}$ $\tan n\theta \rightarrow \frac{\pi}{n}$

$T = 500 + 150 \sin\theta + 150 \sin 2\theta - 100 \cos 3\theta$

time period $= \frac{\text{LCM of } N^r}{\text{HCF of } D^r}$

time period $\frac{2\pi}{1}$ $\frac{2\pi}{2}$ $\frac{2\pi}{3}$

$= \frac{2\pi}{1}$

6/9

mass	r	θ	mr cosθ	mr sinθ	l
10	10	90°	0	100	
5	20	330	86.60	−50	
m_c	10	θ_p	$10 m_p \cos\theta_p$	$10 m_p \sin\theta_p$	

$m_D \sin\theta_D = -5$ — (1)

$86.60 + m_D (10) \cos\theta_D = 0$

$m_D \cos\theta_D = -8.66$ — (ii)

$m_D = \sqrt{(-5)^2 + (-8.66)^2}$ $\quad$ $\tan\theta_D = -5/-8.66$

$\boxed{m_D = 10 \text{ kg}}$ $\quad$ $\boxed{\theta_D = 30}$

$\theta_D = 210°$ from (+x axis)

d)

$F_c = m_D r_D \omega^2 = 3947.84$ N

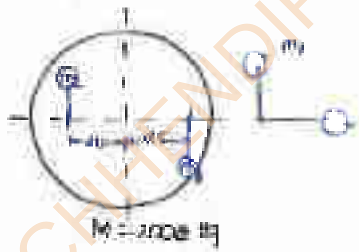
$F_C = F_D$

$3947.84 \times \frac{20}{100} = R \times \frac{40}{100}$

$\boxed{R = 1973.9 \text{ N}} \approx 2$ kN

[e]

mass	r	θ	mr cosθ	mr sinθ
52 kg	10	0°	(52)(10)	0
2000	e	θ	(2000)(e) cosθ	(2000)(e) sinθ
75 kg	10	90	0	(75)(10)



$(52)(10) + (2000)(e)\cos\theta = 0$

$\boxed{e\cos\theta = -0.26}$ — (i)

$(75)(10) + (2000)(e)\sin\theta = 0$

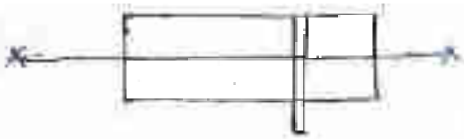
$\boxed{e\sin\theta = -0.375}$

$e^2 = (-0.26)^2 + (-0.375)^2 \Rightarrow \boxed{e = 0.456 \text{ cm}}$

$\tan\theta = \frac{-0.375}{-0.26}$

$\boxed{\theta = 55.26°} \Rightarrow 235.26°$ (from +x axis)

[44]



$r_A = r_B = 0.5$

mass	r	θ	m r cos θ	m r sin θ	l	m r l cos θ	m r l sin θ
(R.P) m_A	0.5	θ_A	$0.5 m_A \cos\theta_A$	$0.5 m_A \sin\theta_A$	0	0	0
4		0°	2	0	0.3	0.6	0
m_B	0.5	θ_B	$0.5 m_B \cos\theta_B$	$0.5 m_B \sin\theta_B$	0.5	$0.5^2 m_B \cos\theta_B$	$0.5^2 m_B \sin\theta_B$

→ $0.5^2 m_B \sin\theta_B = 0$

$\theta_B = 0$ or $180°$

→ $\theta_B = 0 \Rightarrow 0.6 + 0.5^2 m_B \cos 0 = 0$ ← $\theta = 180°$ only possible

$m_B = 2.4$ kg ; $\theta_B = 180 \rightarrow m_B = 2.4$ kg

→ $\Sigma m r \sin\theta = 0$

$(0.5)(2.4)\sin(180) + (0.5) m_A \sin\theta_A = 0$

$m_A \sin\theta_A = 2.4$ — (i)

→ $\Sigma m r \cos\theta = 0$

$2 + (0.5)(2.4)\cos(180) + (0.5) m_A \cos\theta_A = 0$

$m_A \cos\theta_A = 2.4$ →

$2 - 1.2 + (0.5) m_A \cos\theta_A = 0$

$m_A \cos\theta_A = -1.6$ — (ii)

$m_A = 2.66$ kg

$\Sigma m r \sin\theta = 0$

$0.5 m_A \sin\theta_A + 0 + 0.5 m_B \sin\theta_B = 0$ { $\theta_B = 0$

$\theta_A = 0$ or 180

→ $\Sigma m r \cos\theta = 0$

$0.5 m_A \cos\theta_A + 2 + 0.5 m_B \cos\theta_B = 0$

$m_A = 1.6$ kg

mass	r	θ	mr cosθ	mr sinθ
20	15	0°	300	0
25	20	135°	−353.55	353.55
M	30	θ	(30)(M) cosθ	(30)(M) sinθ

$\Rightarrow$ $353.55 + (30)(M)\sin\theta$

$M\sin\theta = -11.67$ — (i)

$\Rightarrow$ $300 - 353.55 + (30)(M)\cos\theta = 0$

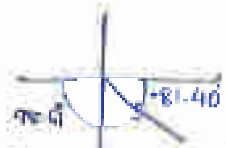
$M\cos\theta = 2.67$ — (ii)

$M = 17.87$ kg m

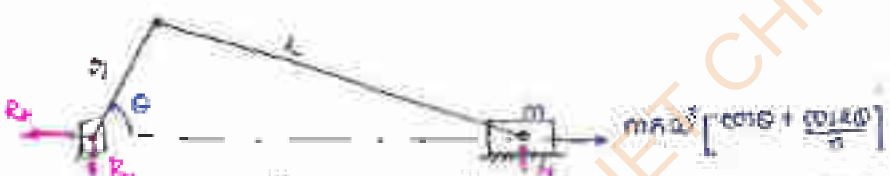
$\tan\theta = \frac{-17.67}{2.67} \Rightarrow \theta = -81.40$

$= 98.59$ (from −x axis)

$= 278.6$ (from +x axis)



→ Balancing of Reciprocating parts :



→ $(F_i)_{net} = mr\omega^2\cos\theta + \frac{mr\omega^2\cos 2\theta}{n}$

(primary inertia force) — primary ; secondary

→ $n = \frac{l}{r}$; $n > 1$

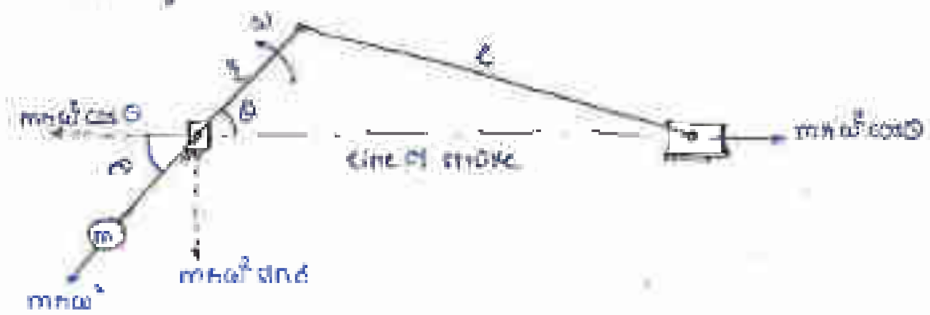
→ Since n is quite large ; so : secondary inertia force smaller in magnitude with respect to primary inertial force & hence it can be neglected.

→ The primary inertia force $mr\omega^2\cos\theta$ results an unbalanced force which changes its magnitude as well as direction as the crank rotates.

→ R_H is known as shaking force.

→ R_V and normal thrust acting on slider will introduce a couple known as shaking couple.

force with the help of balancing mass or counter mass
- It has been observed that F_y is more harmful for the pin with respect to F_x therefore we require partial balancing



- Vertical force is very danger so balancing of reciprocating parts neutral completely is not possible so we go for partially balance.

Balancing mass = B
balancing radius = b

$$B \cdot b = C \cdot m \cdot r$$

where :
- B = balancing mass
- b = balancing mass radius
- c = fraction
- m = mass of reciprocating parts
- r = crank radius

⟹ unbalanced force along line of stroke

$$F_{un,H} = m r \omega^2 \cos\theta - B b \omega^2 \cos\theta$$

$$= m r \omega^2 \cos\theta - c\, m r \omega^2 \cos\theta$$

$$F_{un,H} = (1 - c)\, m r \omega^2 \cos\theta$$

$F_{un,v} = c\, m r \omega^2 \sin\theta$

⇒ Net unbalance force

$R_{net} = \sqrt{F_{un,H}^2 + F_{un,v}^2}$

$= \sqrt{((1-c) m r \omega^2 \cos\theta)^2 + (c\, m r \omega^2 \sin\theta)^2}$

NOTE: R_{net} will be minimum at $c = \frac{1}{2}$

in steam engine $c = \frac{2}{3}$

(3) Effects

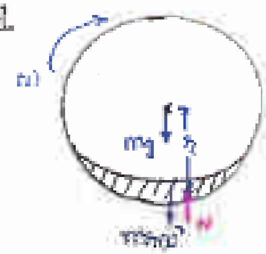
(1) Hammer blow

- The maximum magnitude of unbalanced force perpendicular to the line of stroke is called a hammer blow.

$(F_{un,v})_{max} = m r \omega^2$ ⇒ in complete balancing

NOTE: In order to reduce the unbalance force perpendicular to the line of stroke we did partial balancing.

Wheel



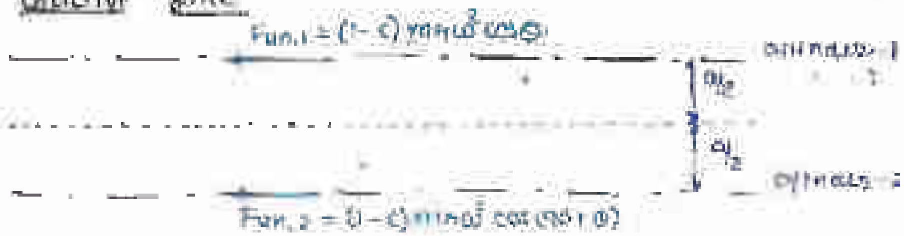
$Mg \pm m r \omega^2 = N$

- The Hammer blow puts a limit on the speed of locomotive.

(2) Effect in coupled locomotive

- In coupled locomotive cranks will be connected perpendicular to each other.

→ Tractive force



$F_{un,net} = F_{un,1} - F_{un,2}$

$F_{un,net} = (1-c)\, m r \omega^2 [\cos\theta - \sin\theta]$

$F_{un,net} = f(\theta)$

$d\theta$

$-\sin\theta - \cos\theta = 0$

$\tan\theta = -1$

$\boxed{\theta = 135^\circ \text{ or } 315^\circ}$

F_{un} @ $\theta = 135^\circ$ $= (1-c)\, m r \omega^2 [\cos 135^\circ - \sin 135^\circ]$

$= -\sqrt{2}\, m r \omega^2 (1-c)$

F_{un} @ $\theta = 315^\circ$ $= +\sqrt{2}\, m r \omega^2 (1-c)$

$\boxed{\text{Tractive force} = \pm\sqrt{2}(1-c)\, m r \omega^2}$

→ Swaying couple

$M = (1-c)\, m r \omega^2 \cos\theta \cdot \frac{a}{2} - (1-c)\, m r \omega^2 \cos(90+\theta) \frac{a}{2}$

$\boxed{M = (1-c)\, m r \omega^2 \frac{a}{2} [\cos\theta + \sin\theta]}$

$M = f(\theta)$

for max^m & min^m

$\frac{dM}{d\theta} = 0$

$\Rightarrow (1-c)\, m r \omega^2 \frac{a}{2} [-\sin\theta + \cos\theta] = 0$

$\tan\theta = 1$

$\boxed{\theta = 45^\circ \text{ or } 225^\circ}$

M @ $\theta = 45^\circ = (1-c)\, m r \omega^2 \frac{a}{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$

$= +\frac{1}{\sqrt{2}} (1-c)\, m r \omega^2 a$

M @ $\theta = 225^\circ = -\frac{1}{\sqrt{2}} (1-c)\, m r \omega^2 a$

$\boxed{\text{Swaying couple} = \pm \frac{a}{\sqrt{2}} (1-c)\, m r \omega^2}$

$F_{primary} = m r \omega^2 \cos\theta$

$F_{secondary} = m r \omega^2 \frac{\cos 2\theta}{n}$

m
r
ω
θ

$F_s = m\left(\frac{r}{4n}\right) 4\omega^2 \cos 2\theta$

$= m\left(\frac{r}{4n}\right)(2\omega)^2 \cos 2\theta$

$F_s = m r' (\omega')^2 \cos 2\theta$

for converting primary → secondary

$m \to m$
$r' = r/4n$
$\omega' = 2\omega$
$\theta' = 2\theta$

→ Rotating can be balance completely whereas Reciprocating masses are partially balance always.

$B \cdot b = m_{rota} \cdot r + c\, m_{rec} \cdot r$

if $b = r$

$B = m_{rota} + c\, m_{rec}$

[13] $m = 10$ kg

$r = 0.1$ m (stroke $= 2r = 0.2$)

$B = 5$ kg

$b = r = 0.1$

$\theta = 30$

$F_{unb} = c\, m r \omega^2 \sin\theta$

$= B b \omega^2 \sin 30 = 5 \times 0.1 \times 10^2 \times \frac{1}{2}$

$F_{un} = 30$ N

$Bb = c\, m r$

[16] $m = 10$ kg

$r = 15$ cm

$c = 0.6$

$\theta = 60$

$\omega = \pi$ rad/s

along line of axis $F_{un,H} = (1-c) m r \omega^2 \cos\theta$ (fraction of mass balanced)

$= (1-0.6)(10)(0.15)(\pi)^2 \cos 60$

$F_{un,H} = 4.5$ N

⇒ Classification of vibration

1) On the basis of excitation

(i) Natural vibration

If the system vibrates due to inherent forces or self weight

Does not require any external forces, is defined as Natural vibration.

(ii) Free vibration

- If an external force is required to produce the vibration in the system & force will not be considered during the analysis of motion is called free vibrations.
- In free vibration, the energy of system may or may not remain conservative.

(iii) Forced vibration

- The vibration due to external excitation force is defined as forced vibration.

(iv) Parametrically excited vibration

(v) Self excited vibration

eg. vocal chord of human body

2) On the basis of degree of freedom

(i) Single D.O.F

(ii) Multi D.O.F

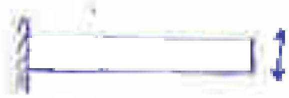
(iii) Infinite DOF

Ex. elastic bodies

3) On the basis of direction of motion

(i) Longitudinal vibration

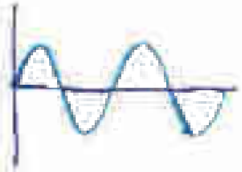
(ii) Transverse vibration



(iii) Torsional vibration

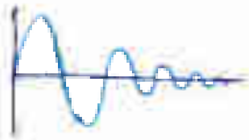
Vibration

Undamped (Energy = const.)



Damped vibration

- viscous damping
- friction (or) columb damping
- Gravitational damping
- Hysteris



Linearization of parameter

⇒ **System Parameter**

1) Interia

- The ability of any body to resist the change is known as interia.
- measure of interia in pure translation is mass when in pure rotation it is mass moment of interia.

pure translation

$$F_{ext} = \frac{d}{dt}(mv)$$

$$F_{ext} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

if m = const.

$$F_{ext} = m\frac{dv}{dt}$$

$$F_{ext} = m a_{cm}$$

$F_I = m \cdot a_{cm}$
$F_I = m \cdot \ddot{x}$

in translation

if 'x' is displacement

$$\frac{dx}{dt} = \dot{x} \quad \text{velocity}$$

$$\frac{d^2x}{dt^2} = \ddot{x} \quad \text{accn}$$

in rotation

θ

$$\frac{d\theta}{dt} = \dot{\theta}$$

$$\frac{d^2\theta}{dt^2} = \ddot{\theta}$$

of inertia (MOI) (which is distribution of mass about axis of rotation)

Rigid Rotation

$T_{ext} = \frac{d}{dt}(m v_G)$

$T_i = I \ddot{\theta}$

external torque — Mass M.O.I

⇒

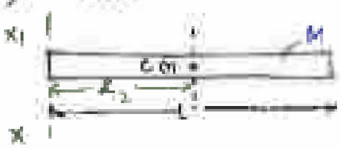
1) Disc

$I_G = \frac{MR^2}{2}$

2) concentrated mass

$I_P = m l^2$

3) Rod



$I_G = \frac{Ml^2}{12}$

$I_{x_1x_1} = I_G + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12} + \frac{ml^2}{4}$

$I_{x_1x_1} = \frac{ml^2}{3}$

4)

$I_{x_1x_1} = I_G + Mr^2$

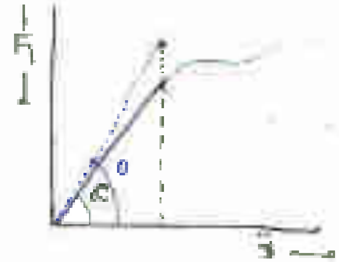
$= \frac{mr^2}{2} + mr^2$

$I_{x_1x_1} = \frac{3}{2} mr^2$

5)

$I_{x_2x_2} = I_G + ma^2$

$\tan\theta = m$



mass : Mass is the slope of F_i vs $\ddot{x}$ diagram upto Linear Region

∴ mass ↑ α ↑ F_i ↑

- $m_1 > m$
- $\theta_1 > \theta$
- $F_1 > F$

→ This is a region we always have smaller link of drive in belt drive. In gear - pinion mechanism, pinion is driven → having lower inertia → low starting torque

Restoration

- Every body comes back to its initial position, this ability is known as restoration
- to represent restoration characteristic of any system we use springs

Hooke's law :

$F_{spring} \propto -x$

(−ve) sign

Explanation.

→ Negative sign indicates that spring force will always be opposite to the displacement

$F_s = -kx$ → $k = \frac{F_s}{x}$

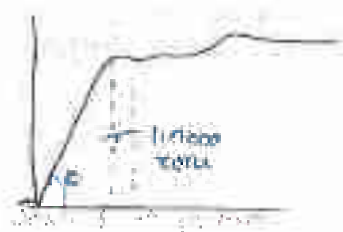
If $x = 1$ unit

$k = F_s$

The amount of force req^d to produce unit deflection is known as stiffness.

→ k ↑ F_s ↑ chances of failure ↓

$\tan\alpha = k$

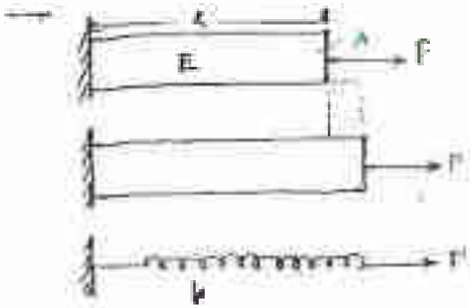


Where k = stiffness
= spring constant
= spring rate

unit = N/m

Q Ob $K_{eq} = \frac{T}{\theta}$

unit - $\frac{N\text{-}m}{rad}$



In solids

$\sigma \propto \epsilon$ (P-L)

$\sigma = E\epsilon$

$\frac{P}{A} = E \frac{\delta L}{L}$

$P = \left(\frac{AE}{L}\right) \delta L$

Axial stiffness $= \frac{AE}{L}$

In spring

$F_s \propto x$

$F_s = kx$

Loading	Geometrical property	Material property	Rigidity	Stiffness = $\frac{\text{Rigidity}}{\text{length}}$
Axial	A	E	AE axial rigidity	Axial stiffness $= \frac{AE}{L}$
flexural	I	E	EI flexural rigidity	flexural stiffness $= f\left(\frac{EI}{L}\right)$
Torsion	J	G	GJ	$\frac{GJ}{L}$

IMP → stiffness $\propto \frac{1}{\text{length}}$

i.e. length ↑
k ↓
rigidity ↓
chances of failure ↑

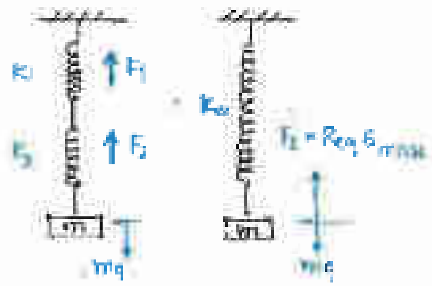
→ If the length of k are into ratio of m : n then their stiffness will be in a ratio n : m

No. of turns

equal ... than springs of equal stiffness '2k' & '3k'.

⇒ Spring connection:

i) Series connection



$F_1 = F_2 = mg$ (Forces are same)

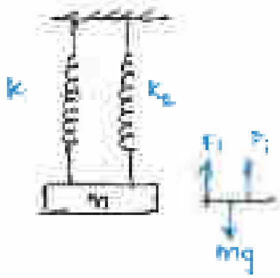
$\delta_{max} = \delta_1 + \delta_2$ (deflection is cumulative)

$$\frac{mg}{K_{eq}} = \frac{F_1}{K_1} + \frac{F_2}{K_2}$$

$$\frac{1}{K_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\frac{1}{K_{eq}} = \sum_{i=1}^{n} \frac{1}{K_i}$$

ii) Parallel connection



1) Mass remains horizontal

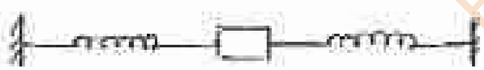
$F_1 + F_2 = mg$ (Forces are cumulative)

$\delta_1 = \delta_2 = \delta_{max}$

$K_1\delta_1 + K_2\delta_2 = K_{eq}\,\delta_{max}$

$$K_{eq} = K_1 + K_2$$

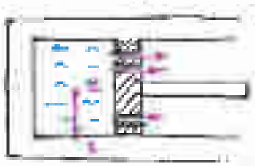
$$K_{eq} = \sum_{i=1}^{n} K_i$$

iii)  → parallel connection

parallel conn are more rigid → because combined stress incr

(3) ⇒ Damping:

i) Viscous damping:



Newton's law of viscosity

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \left[\frac{u_2 - u_1}{y_2 - y_1}\right]$$

At no slip condition $u_1 = 0$

y_2

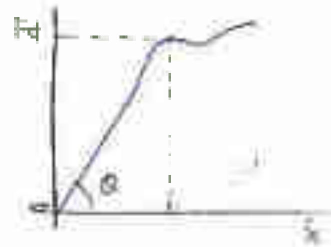
$\tau \propto \frac{u}{y}$

$\tau \propto$ velocity of fluid

$F_d \propto$ velocity of fluid

→ $F_d \propto \dot{x}$

$F_d = c\dot{x}$ where c = damping coefficient



In translation:

$c = \frac{F_d}{\dot{x}}$ unit $\frac{N}{(m/s)} = \frac{N\cdot s}{m}$

In Rotation

$c_{eq} = \frac{T_d}{\dot{\theta}}$ unit $\frac{N\cdot m}{rad/s} = \frac{N\cdot m\cdot sec}{rad}$

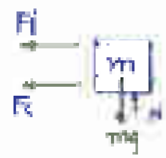
(b)

→ Equilibrium position :

- The position about which system vibrate

[■] Equation of motion in single d.o.f. / undamped free vibration

case-(i)



Assumption: spring is massless.

$F_i + F_s = 0$

$m\ddot{x} + kx = 0$

$\ddot{x} + \frac{k}{m}x = 0$ compare with $\ddot{x} + \omega_n^2 x = 0$

→ $\omega_n = \sqrt{\frac{k}{m}}$

where ω_n = natural angular frequency of undamped system

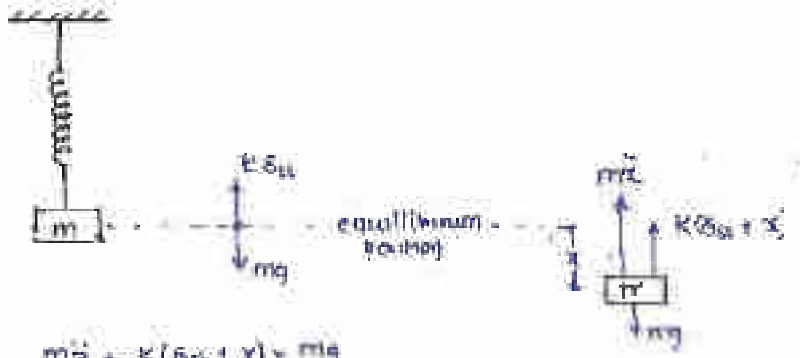
→ $\omega_n = 2\pi f_n$

f_n = linear frequency Hz or sec

→ Time period $T = \frac{1}{f_n}$ unit T → sec

Time period: Time required to complete one cycle is called time period.

Case-(ii):



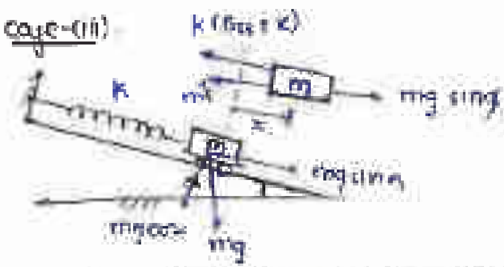
$$m\ddot{x} + K(\delta_{st} + x) = mg$$

$$m\ddot{x} + K\delta_{st} + Kx = mg$$

$$\boxed{\ddot{x} + \frac{k}{m}x = 0}$$

where $\boxed{\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta_{st}}}}$

Case-(iii):



At initially $\boxed{mg \sin\alpha = K\delta_{st}}$

$\rightarrow m\ddot{x} + K(\delta_{st} + x) = mg\sin\alpha$

$\rightarrow m\ddot{x} + K\delta_{st} + Kx = mg\sin\alpha$

$$\boxed{\ddot{x} + \frac{K}{m}x = 0}$$

where $\boxed{\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g\sin\alpha}{\delta_{st}}}}$

$\therefore \sqrt{\frac{k}{m}} = \sqrt{\frac{mg\sin\alpha}{m\delta_{st}}}$

$\Rightarrow$ condition of equation of motion

$$\ddot{x} + \omega_n^2 x = 0$$

$$\rightarrow \frac{d^2x}{dt^2} + \omega_n^2 x = 0$$

$[D^2 + \omega_n^2]x = 0$

→ soln of x = $f(t)$

soln = CF + PI (PI = 0 because right part is zero)

CF.

$D^2 + \omega_n^2 = 0$

$\Rightarrow D^2 = -\omega_n^2$

$$\boxed{D = \pm i\,\omega_n}$$

→ $x = A\cos\omega_n t + B\sin\omega_n t$ { in vibration it is still follow

$A = X\sin\phi$

$B = X\cos\phi$

$x = X\sin\phi\cos\omega_n t + X\cos\phi\sin\omega_n t$

$$\boxed{x = X\sin(\omega_n t + \phi)}$$ ← Displacement eqn

Velocity eqn

$\dot{x} = X\omega_n\cos(\omega_n t + \phi)$

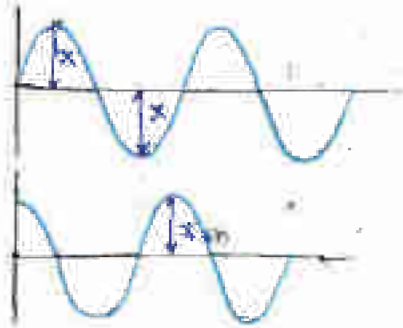
Accn eqn

$\ddot{x} = -X\omega_n^2\sin(\omega_n t + \phi)$

→ Analogy b/w translation & rotation

Pure translation	Rotation
→ $m\ddot{x} + Kx = 0$	→ $I\ddot{\theta} + K_{eq}\theta = 0$
$\boxed{\omega_n = \sqrt{\frac{K}{m}}}$	K_{eq} $\boxed{\omega_n = \sqrt{\frac{K_{eq}}{I}}}$ ← about centre of rotation
→ Displacement $x = X\sin(\omega_n t + \phi)$	→ $\theta = \Theta\sin(\omega_n t + \phi)$
Velocity $\dot{x} = X\omega_n\cos(\omega_n t + \phi)$	$\dot{\theta} = \Theta\omega_n\cos(\omega_n t + \phi)$
Accn $\ddot{x} = -X\omega_n^2\sin(\omega_n t + \phi)$	$\ddot{\theta} = -\Theta\omega_n^2\sin(\omega_n t + \phi)$
$x_{max} = X$	$\theta_{max} = \Theta$
$\dot{x}_{max} = X\omega_n$	$\dot{\theta}_{max} = \Theta\omega_n$
$\ddot{x}_{max} = -X\omega_n^2$	$\ddot{\theta}_{max} = \Theta\omega_n^2$

$x = X \sin(\omega_n t + \phi)$



$\dot{x} = X\omega_n \cos(\omega_n t + \phi)$

NOTE –
There is 90° phase lag between displacement & velocity vector, acc^n ∝ – displacement ; which indicate the motion is SHM

→ Energy Method :

T.E of system = const

$\frac{d}{dt}$ [T.E of system] = 0

$\frac{d}{dt}\left[\frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2\right] = 0$

(K.E of mass) (P.E of spring)

$\Rightarrow \frac{1}{2}(2m\dot{x}\ddot{x}) + \frac{1}{2}(2kx\dot{x}) = 0$

$m\ddot{x} + kx = 0$

$$\boxed{\ddot{x} + \frac{k}{m}x = 0}$$ ← valid for only undamped system

→ Rayleigh's Method :

– If the system is undamped, then T.E of system = const

i.e. max^m energy at equilibrium = max^m energy at extreme position

$\Rightarrow \frac{1}{2} m\dot{x}^2_{max} = \frac{1}{2} kx^2_{max}$

$\Rightarrow m\dot{x}^2_{max} = kx^2_{max}$

$\Rightarrow m(X\omega_n)^2 = k(X)^2$

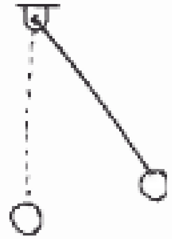
$\omega_n = \sqrt{\frac{k}{m}}$

At mean position V = max
Extreme position V = 0

NOTE:

→ $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\Rightarrow \lim_{\theta \to 0} \sin\theta = \theta$



→ $\lim_{\theta \to 0} \cos\theta = 1$

→ $\lim_{\theta \to 0} (1-\cos\theta) = \frac{\theta^2}{2}$ $\left\{ \cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots \right.$

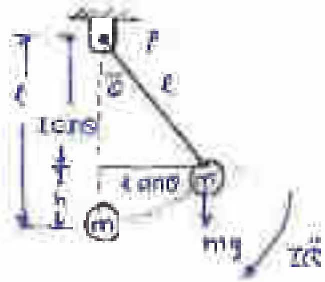
case - II :

let c.w. couple +ve.

$I_p \ddot{\theta} + mgl\sin\theta = 0$

$I_p \ddot{\theta} + mgl\theta = 0$

$\ddot{\theta} + \left(\frac{mgl}{I_p}\right) = 0$



$\omega_n = \sqrt{\frac{mgl}{I_p}}$ as $\boxed{I_p = ml^2}$

$\frac{2\pi}{T} = \sqrt{\frac{mgl}{ml^2}} = \sqrt{\frac{9.81}{l}}$ $\boxed{\omega_n = \frac{2\pi}{T}}$

$\frac{2\pi}{0.5} = \sqrt{\frac{9.81}{l}}$

$\boxed{l = 62\ mm}$

Energy method

$\frac{d}{dt}$ (TE of system) $= 0$

→ $\frac{d}{dt}$ (KE + PE) $= 0$

→ $\frac{d}{dt}\left(\frac{1}{2} I_p \dot{\theta}^2 + mgh\right) = 0$

→ $\frac{d}{dt}\left(\frac{1}{2} I_p \dot{\theta}^2 + mg(l - l\cos\theta)\right) = 0$

→ $\frac{d}{dt}\left(\frac{1}{2} I_p \dot{\theta}^2 + mgl(1-\cos\theta)\right) = 0$

→ $\frac{d}{dt}\left(\frac{1}{2} I_p \dot{\theta}^2 + mgl \frac{\theta^2}{2}\right) = 0$

— I_p due to mass m; some displacement is come (happened) then the m will not come.

$$I_P\ddot{\theta} + mg\ell\theta = 0$$

⇒ **Rayleigh's method**

max KE = max P.E

$$\Rightarrow \frac{1}{2} I_P \dot{\theta}_{max}^2 = mg\ell\,[1-\cos\theta]_{max}$$

$$\Rightarrow \frac{1}{2} I_P \dot{\theta}_{max}^2 = mg\ell\,\frac{\theta_{max}^2}{2}$$

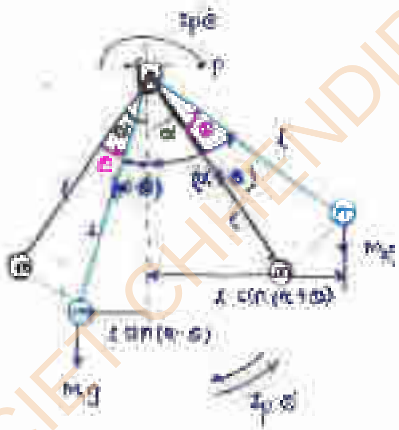
$$\Rightarrow I_P(\theta\omega_n)^2 = mg\ell\,\theta^2$$

$$\omega_n = \sqrt{\frac{mg\ell}{I_P}} = \sqrt{\frac{mg\ell}{m\ell^2}} = \sqrt{\frac{g}{\ell}}$$

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{\ell}{g}}$$

(ℓ = 82.1mm)

Q.2 c.w. (+ve)



$$\Rightarrow I_P\ddot{\theta} + m_2 g(\ell\sin(\alpha+\theta)) - m_1 g(\ell\sin(\alpha-\theta)) = 0$$

$$\Rightarrow I_P\ddot{\theta} + m_2 g\ell(\alpha+\theta) - m_1 g\ell(\alpha-\theta) = 0$$

$$\Rightarrow I_P\ddot{\theta} + mg\ell\alpha + mg\ell\theta - mg\ell\alpha + mg\ell\theta = 0$$

$$I_P\ddot{\theta} + 2mg\ell\theta = 0$$

$$\Rightarrow I_P\ddot{\theta} - mg\ell\sin(\alpha-\theta) + mg\ell\sin(\alpha+\theta) = 0$$

$$\Rightarrow I_P\ddot{\theta} + mg\ell\,[\sin(\alpha+\theta) - \sin(\alpha-\theta)] = 0$$

$$\Rightarrow I_P\ddot{\theta} + mg\ell\left[2\cos\left(\frac{\alpha+\theta+\alpha-\theta}{2}\right)\sin\left(\frac{\alpha+\theta-\alpha+\theta}{2}\right)\right] = 0$$

$$\Rightarrow I_P\ddot{\theta} + 2mg\ell\,[\cos\alpha\cdot\sin\theta] = 0 \qquad | \; S - S = 2CS$$

$$I_P\ddot{\theta} + 2mg\ell\cos\alpha\,\theta = 0$$

where $I = m_1\ell^2 + m_2\ell^2 = 2m\ell^2$

$$\ddot{\theta} + \frac{2mg\ell\cos\alpha}{2m\ell^2}\theta = 0$$

$$\omega_n = \sqrt{\frac{g\cos\alpha}{\ell}}$$

12)

$F = 5000$ N

$\delta t = 10^{-4}$ sec.

$m = 1$ kg

$K = 10$ kN/m

initially rest

→ impulse force = change in momentum

$$F\,\delta t = (mv)_f - (mv)_i \quad \{ v_i = 0 \text{ initially rest}$$

$$5000 \times 10^{-4} = 1 \times v_f$$

$$\boxed{v_f = 0.5 \text{ m/s}} \leftarrow v_{max}$$

→ $$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{10 \times 10^3}{1}}$$

$$\boxed{\omega_n = 100 \text{ rad/s}}$$

→ $$\dot{x}_{max} = X\,\omega_n$$

$$0.5 = 100\,X$$

$$\boxed{X = 5 \text{ mm}}$$

[4] ω_n same at moon and earth.

[5]

$$I_P\ddot{\theta} + F_s\left(\frac{l}{2}\cos\theta\right) = 0$$

$$\Rightarrow I_P\ddot{\theta} + \left(K\frac{l}{2}\sin\theta\right)\left(\frac{l}{2}\cos\theta\right) = 0$$

$$\Rightarrow I_P\ddot{\theta} + K\left(\frac{l}{2}\theta\right)\frac{l}{2} = 0$$

$$\Rightarrow I_P\ddot{\theta} + \frac{Kl^2}{4}\theta = 0$$

$$\ddot{\theta} + \frac{Kl^2}{4I_P} = 0$$

$$I_P = \frac{ml^2}{12}$$

$$\omega_n = \sqrt{\frac{Kl^2}{4I_P}} = \sqrt{\frac{Kl^2}{\frac{ml^2}{12}\cdot 4}}$$

$$= \sqrt{\frac{3 \times 800}{1}}$$

$$\boxed{\omega_n = 49 \text{ rad/s}}$$

$$\frac{d}{dt}(KE + PE) = 0$$

$$\frac{d}{dt}\left(\frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2\right) = C$$

$$\frac{d}{dt}\left[\frac{1}{2} I_p \dot{\theta}^2 + \frac{1}{2} k x_c^2\right] = 0$$

$$\frac{d}{dt}\left\{ I_p \dot{\theta}^2 + k\left(\frac{l}{2}\theta\right)^2 \right\} = 0$$

$$\Rightarrow \frac{d}{dt}\left[I_p \dot{\theta}^2 + \frac{k\theta^2 l^2}{4} \right] = 0$$

$$\Rightarrow I_p(2\dot{\theta}\ddot{\theta}) + \frac{kl^2}{4}(2\theta\dot{\theta}) = C$$

$$\boxed{I_p\ddot{\theta} + \frac{kl^2}{4}\theta = 0}$$

→ Rayleigh method

$$(K.E)_{max} = (P.E)_{max}$$

$$\frac{1}{2} I_p \dot{\theta}^2_{max} = \frac{1}{2} k x^2_{max}$$

$$\frac{1}{2} I_p \dot{\theta}^2_{max} = \frac{1}{2} k \left(\frac{l}{2}\theta\right)^2$$

$$I_p(\dot{\theta}_{max})^2 = \frac{kl^2}{4}\theta^2$$

$$I_p(\theta\omega_n)^2 = \frac{kl^2}{4}\theta^2$$

$$\boxed{\omega_n = \sqrt{\frac{kl^2}{4 I_p}}}$$

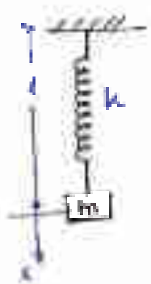
6) $f_n = \frac{1}{2\pi}\sqrt{\frac{ka^2 - wb}{I_o}}$

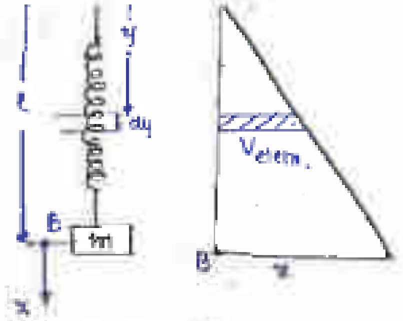
for fn' $\boxed{f_n > 0}$

$$\sqrt{\frac{ka^2 - wb}{I_o}} > 0$$

$$ka^2 - wb > 0$$

$$wb < ka^2 \rightarrow \boxed{b < ka^2/w}$$

9) 



$$\frac{V_{elem}}{\dot{x}} = \frac{y}{\ell}$$

let mass per unit length of spring = γ

$M_{spring} = \gamma L$

Energy method :

TE of system = KE of mass + KE of spring + PE of spring

K.E of spring = $\int d(KE)_{element}$

$dm|_{element} = \gamma dy$

$V_{element} = \frac{\dot{x} y}{\ell}$

$$KE \text{ of spring} = \int \frac{1}{2} (dm)_{element} (V_{element})^2$$

$$= \int \frac{1}{2} \gamma dy \left(\frac{\dot{x} y}{\ell}\right)^2$$

$$= \frac{\gamma \dot{x}^2}{2\ell^2} \int_0^{\ell} y^2 dy$$

$$= \frac{\gamma \dot{x}^2}{2\ell^2} \left[\frac{y^3}{3}\right]_0^{\ell} = \frac{\gamma}{2} \dot{x}^2 \cdot \frac{\ell}{3}$$ but $\gamma L = M_{spring}$

$$\boxed{KE \text{ of spring} = \frac{1}{6} m_s \dot{x}^2}$$

→ TE of system $= \frac{1}{2} m\dot{x}^2 + \frac{1}{6} m_s \dot{x}^2 + \frac{1}{2} kx^2$

$$\frac{d}{dt}(TE) = \frac{1}{2} m(2\dot{x}\ddot{x}) + \frac{1}{6} m_s (2\dot{x}\ddot{x}) + \frac{1}{2}(k \, 2x\dot{x}) = 0$$

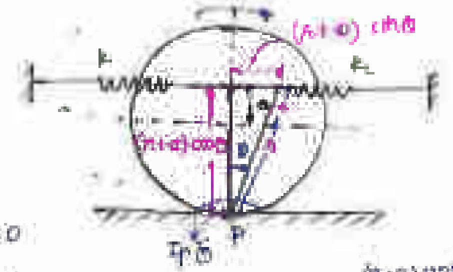
$$m\ddot{x} + \frac{m_s}{3}\ddot{x} + kx = 0$$

$$\left(m + \frac{m_s}{3}\right)\ddot{x} + kx = 0$$

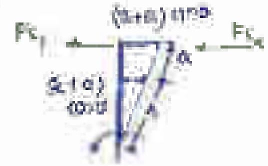
$$\ddot{x} + \frac{k}{\left(m + \frac{m_s}{3}\right)} x = 0$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m + \frac{m_s}{3}}}}$$

→ Rolls w/o slipping so at centre of rotation about P.



$I_P\ddot\theta + F_{s_2}(r+a)\cos\theta$
$+ F_{s_1}(r+a)\cos\theta = 0$

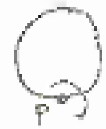


$\Rightarrow I_P\ddot\theta + [k_2(r+a)\theta](r+a)(1)$
$+ [k_1(r+a)\theta](r+a)(1) = 0$

$\Rightarrow I_P\ddot\theta + k(r+a)^2\theta + k(r+a)^2\theta = 0$

$\Rightarrow I_P\ddot\theta + 2k(r+a)^2\theta = 0$

$\ddot\theta + \left[\frac{2k(r+a)^2}{I_P}\right] = 0$



$I_P = \frac{3}{2}mr^2$

$mr^2 + \frac{mr^2}{2}$

$\omega_n = \sqrt{\frac{2k(r+a)^2}{\frac{3}{2}mr^2}} = \sqrt{\frac{4k(r+a)^2}{3mr^2}}$

$\omega_n = 500$ rad/s & $f =$

→ Energy method

$\frac{d}{dt}\left[\frac{1}{2}I_P\dot\theta^2 + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2\right] = 0$

$\frac{d}{dt}\left[\frac{1}{2}I_P\dot\theta^2 + k_1x^2\right] = 0$

$\frac{d}{dt}\left[I_P\dot\theta^2 + 2k_1x^2\right] = 0$

$I_P\dot\theta^2 + 2k[(r+a)\theta]^2 = 0$

$2I_P\dot\theta\ddot\theta + 2k(r+a)^2\theta(2)\dot\theta = 0$

(a) below pt

$\ddot\theta + \frac{4k(r+a)^2}{2I_P} = 0$

$I_P = \frac{3}{2}mr^2$

$\omega_n = \sqrt{\frac{4k(r+a)^2}{2\left(\frac{3}{2}mr^2\right)}}$

$\omega_n = \sqrt{\frac{4k(r+a)^2}{3mr^2}}$

[16]

displacement = a+d

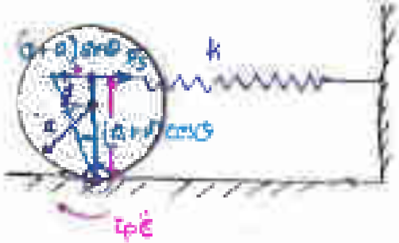
$I_P\ddot{\theta} + F_s (a+d)\cos\theta = 0$

$I_P\ddot{\theta} + K(a+d)\theta\,(a+d) = 0$

$I_P\ddot{\theta} + K(a+d)^2\theta = 0$

$\ddot{\theta} + \frac{K(a+d)^2}{I_P}\theta = 0$

$\omega_n = \sqrt{\frac{2K(a+d)^2}{3ma^2}}$



$I_P = \frac{3}{2}ma^2$

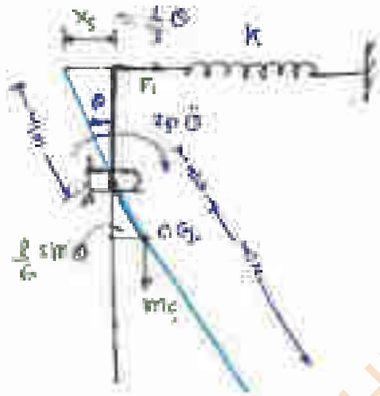
[9]

$\therefore I_P\ddot{\theta} + F_s\frac{l}{3}\cos\theta + mg\frac{l}{6}\sin\theta = 0$

$\Rightarrow I_P\ddot{\theta} + (Kx_s)\frac{l}{3} + mg\frac{l}{6}\theta = 0$

$\Rightarrow I_P\ddot{\theta} + K\left(\frac{l}{3}\theta\right)\frac{l}{3} + mg\frac{l}{6}\theta = 0$

$\Rightarrow I_P\ddot{\theta} + \frac{Kl^2}{9}\theta + \frac{mgl}{6}\theta = 0$



$\omega_n = \sqrt{\frac{\frac{Kl^2}{9} + \frac{mgl}{6}}{I_P}}$

dist from (C.G. to where we have to find)

$I_P = I_{CG} + m\left(\frac{l}{6}\right)^2$

$I_{CG} = \frac{ml^2}{12}$

$= \frac{ml^2}{12} + \frac{ml^2}{36}$

$= \frac{3ml^2}{36} + \frac{ml^2}{36} = \frac{ml^2}{9}$

$\omega_n = \sqrt{\frac{Kl^2}{9\left(\frac{ml^2}{9}\right)} + \frac{mgl}{6\left(\frac{ml^2}{9}\right)}}$

$$\boxed{\omega_n = \sqrt{\frac{3g}{2l} + \frac{K}{m}}}$$

$$\frac{d}{dt}(T.E) = 0$$

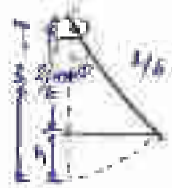
$$\Rightarrow \frac{d}{dt}\left[\frac{1}{2} I_p \dot{\theta}^2 + \frac{1}{2} k x_s^2 + mgh\right] = 0$$

$$\Rightarrow \frac{d}{dt}\left[\frac{1}{2} I_p \dot{\theta}^2 + \frac{1}{2} k \left(\frac{\ell}{3}\theta\right)^2 + mg\left(\frac{\ell}{6}\frac{\theta^2}{2}\right)\right] = 0$$

$$\Rightarrow \frac{d}{dt}\left[\frac{1}{2} I_p \dot{\theta}^2 + \frac{1}{2} k \frac{\ell^2}{9}\theta^2 + \frac{1}{2} mg \frac{\ell}{6}\theta^2\right] = 0$$

$$I_p(2\dot{\theta}\ddot{\theta}) + \frac{k\ell^2}{9}(2\theta\dot{\theta}) + mg\frac{\ell}{6}(2\theta)\dot{\theta} = 0$$

$$I_p\ddot{\theta} + \left(\frac{k\ell^2}{9}\right)\theta + \left(\frac{mg\ell}{6}\right)\theta$$



$$h = \frac{\ell}{6} - \frac{\ell}{6}\cos\theta$$
$$= \frac{\ell}{6}(1-\cos\theta)$$
$$= \frac{\ell}{6}\frac{\theta^2}{2}$$

$$\boxed{\omega_n = \sqrt{\frac{\frac{k\ell^2}{9} + \frac{mg\ell}{6}}{I_p}}}$$

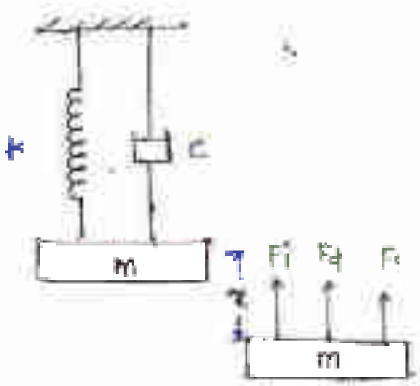
10. $\omega = 5$ rad/s
$x = 10$ cm

X = max. displacement or initial

$x = X \sin(\omega_n t + \phi)$

$\boxed{X = 10 \text{ cm}}$

Single D.O.F. / Damped / free vibration :



$$F_i + F_d + F_s = 0$$

$$\Rightarrow m\ddot{x} + c\dot{x} + kx = 0$$

$$\Rightarrow \boxed{\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0}$$

$$\Rightarrow D^2x + \frac{c}{m}Dx + \frac{k}{m}x = 0$$

where $D = \frac{d}{dt}$

let $x = f(t)$

so $x = C \cdot e^{pt}$

$$\boxed{D^2 + \frac{c}{m}D + \frac{k}{m} = 0}$$

$$D_{1,2} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - \frac{4k}{m}}}{2}$$

$$= -\frac{c}{2m} \pm \frac{1}{2}\sqrt{\left(\frac{c}{m}\right)^2 - \frac{4k}{m}}$$

$$\boxed{D_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}}$$

→ Damping ratio / Damping factor ... (ξ)

$$\because \quad \xi = \sqrt{\frac{(c/2m)^2}{(k/m)}} = \sqrt{\frac{(c/2m)^2}{\omega_n^2}}$$

$$\boxed{\xi = \frac{c}{2m\omega_n}} \quad ; \quad \boxed{\xi = \frac{c}{2\sqrt{km}}}$$

→ $\xi = 0$: undamped system

$\xi > 0$: damped system

- $\xi < 1$: under-damping
- $\xi = 1$: critical damping
- $\xi > 1$: over damping

→ $$\boxed{\xi = \frac{c}{c_c} = \frac{\text{Actual damping coefficient}}{\text{critical damping coefficient}}}$$

→ If $\xi = 0.4$ then it says Actual damping is 40% of critical damping

$$\therefore \quad \xi = \frac{c}{2m\omega_n}$$

$$\therefore \quad 1 = \frac{c_c}{2m\omega_n}$$

$$\boxed{c_c = 2m\omega_n} \quad ; \quad \boxed{c_c = 2\sqrt{km}}$$

So, $$D_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$= -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$\left\{ \xi = \frac{c}{2m\omega_n} \right.$

$\frac{c}{2m} = \xi\omega_n$

$\rightarrow D_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

$D_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{(-1)(1-\xi^2)}$

$$\boxed{D_{1,2} = -\xi\omega_n \pm i\,\omega_n\sqrt{1-\xi^2}}$$

$$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$$

damped natural angular frequency

$$\boxed{D_{1,2} = -\xi\omega_n \pm i\omega_d}$$

Soln. $x = e^{-\xi\omega_n t}[A\cos\omega_d t + B\sin\omega_d t]$

let $A = X\sin\phi$

$B = X\cos\phi$

$x = e^{-\xi\omega_n t}[X\sin\phi\cdot\cos\omega_d t + X\cos\phi\cdot\sin\omega_d t]$

$$\boxed{x = X e^{-\xi\omega_n t}\sin(\omega_d t + \phi)}$$ ← Displacement eqn

repetitive term

$$\rightarrow \boxed{\text{Amplitude} = X e^{-\xi\omega_n t}}$$

exponential decrement

$\therefore x = X e^{-\xi\omega_n t}\sin(\omega_d t + \phi)$

@ $t = 0$

$X_0 = X e^{-0}\sin(0 + \phi)$

$$\boxed{X_0 = X\sin\phi}$$

@ $t = T_d$

$X_1 = X e^{-\xi\omega_n T_d}\sin(\omega_d T_d + \phi)$

$= X e^{-\xi\omega_n \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}}\sin\left(\frac{2\pi}{T_d}T_d + \phi\right)$

$T_d = \frac{2\pi}{\omega_d}$

$= \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$

$$\boxed{X_1 = X e^{-\delta}\sin\phi}$$

$$\boxed{\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

@ $t = 2T_d$

$x = X e^{-\xi\omega_n 2T_d}\sin[\omega_d(2T_d) + \phi]$

$= X e^{-\frac{\xi\omega_n(2\pi)2}{\omega_n\sqrt{1-\xi^2}}}\sin[4\pi + \phi]$

$$x_2 = X e^{-2\delta} \sin\phi$$

$$x_0 = X \sin\phi$$
$$x_1 = X e^{-\delta} \sin\phi$$
$$x_2 = X e^{-2\delta} \sin\phi$$
$$x_3 = X e^{-3\delta} \sin\phi$$
$$\vdots$$
$$x_n = X e^{-n\delta} \sin\phi$$

⇒ Ratio of successive Amplitude

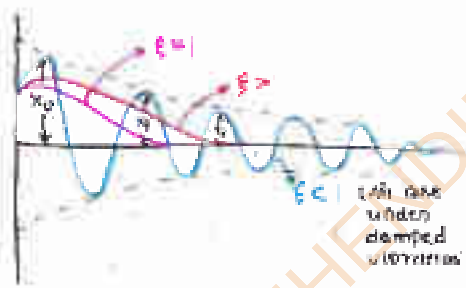
$$\frac{x_0}{x_1} = \frac{X \sin\phi}{X e^{-\delta} \sin\phi} = e^{\delta}$$

$$\frac{x_1}{x_2} = \frac{X e^{-\delta} \sin\phi}{X e^{-2\delta} \sin\phi} = e^{\delta}$$

$$\frac{x_2}{x_3} = \frac{X e^{-2\delta} \sin\phi}{X e^{-3\delta} \sin\phi} = e^{\delta}$$

$$\vdots$$

$$\frac{x_n}{x_{n+1}} = \frac{X e^{-n\delta} \sin\phi}{X e^{-(n+1)\delta} \sin\phi} = e^{\delta}$$



→ $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$ = const {if ξ const}

- The ratio of two successive amplitude is const.

⇒ Logarithmic decrement

$$\frac{x_0}{x_n} = \frac{x_0}{x_1} \cdot \frac{x_1}{x_2} \cdot \frac{x_2}{x_3} \cdots \frac{x_{n-1}}{x_n}$$
$$= e^{\delta} \cdot e^{\delta} \cdots e^{\delta}$$

$$\frac{x_0}{x_n} = e^{n\delta}$$

$$\log_e\left(\frac{x_0}{x_n}\right) = \log_e e^{n\delta}$$

$$\delta = \frac{1}{n} \log_e\left(\frac{x_0}{x_n}\right)$$

→ system will vibrate periodically & it will take infinite time to attain amplitude become zero

→ Critical damping
- It is smallest possible damping for this system will be not vibrate at all

CBD [1]

$$I\ddot{\theta} + T_s - mg\ell\sin\theta = 0$$

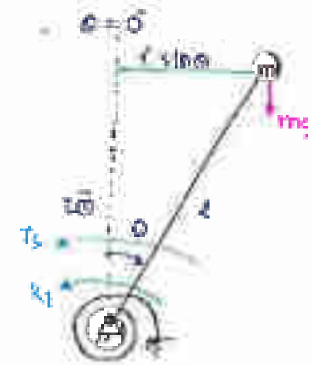
$$I\ddot{\theta} + k_t\theta - mg\ell\sin\theta = 0$$

$$\boxed{I\ddot{\theta} + (k_t - mg\ell)\sin\theta = 0}$$

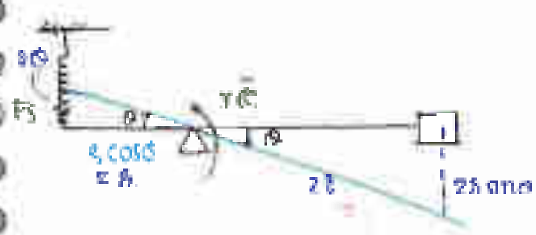
$$I\ddot{\theta} + (k_t - mg\ell)\theta = 0$$

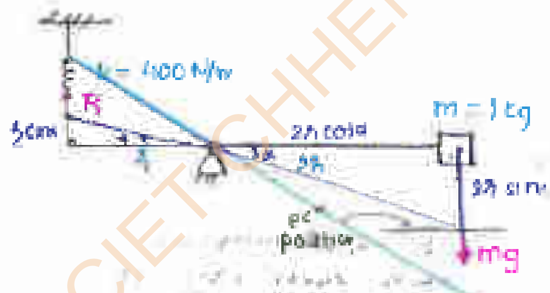
$$I_{eq} = m\ell^2$$

$$k_{eq} = k_t - mg\ell$$



[14]





$$I\ddot{\theta} + F_s(h\cos\theta) = 0$$

$$I\ddot{\theta} + k(h\theta)(h) = 0$$

$$I\ddot{\theta} + kh^2\theta = 0$$

$$\omega_n = \sqrt{\frac{kh^2}{m(2h)^2}} = \sqrt{\frac{k}{4m}} = \sqrt{\frac{400}{4}}$$

$$F_s h = mg\,2h$$

$$F_s = 2mg$$

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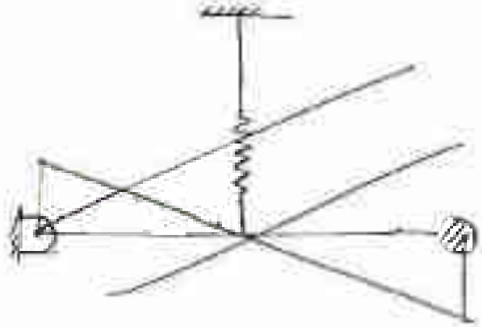
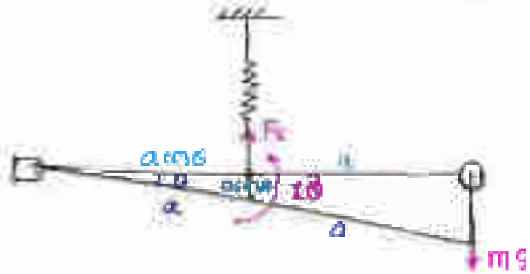


16.

$\frac{mg}{2a} = \frac{F_s}{a}$

$F_s = \frac{mg}{2}$



$\rightarrow I\ddot{\theta} + F_s \cdot (a\cos\theta) = 0$

$\rightarrow I\ddot{\theta} + K(a\theta)(a) = 0$

$\rightarrow I\ddot{\theta} + Ka^2\theta = 0$

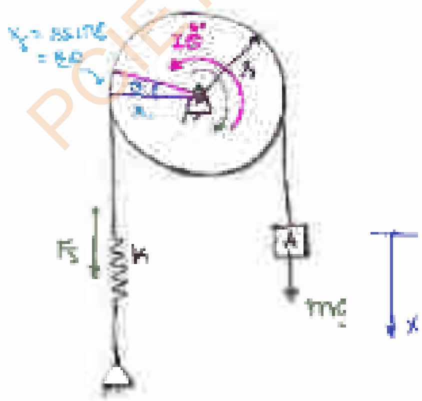
$\boxed{I = m(2a)^2}$

$\omega_n = \sqrt{\frac{Ka^2}{I}}$

$= \sqrt{\frac{Ka^2}{4ma^2}} = \sqrt{\frac{K}{4m}}$

$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{4m}}$

17. If some displacement is given due to weight M then in final eqn mg will not come.



$\rightarrow I\ddot{\theta} + F_s(R) = 0$

$\rightarrow I\ddot{\theta} + (Kx_s)(R) = 0$

$\rightarrow I\ddot{\theta} + (KR\theta)R = 0$

$\rightarrow I\ddot{\theta} + KR^2\theta = 0$

$\ddot{\theta} + \frac{KR^2}{I} = 0$

$\omega_n = \sqrt{\frac{KR^2}{I}} = \sqrt{\frac{KR^2}{(mR^2 + \frac{MR^2}{2})}}$

I_{total} = due to mg + due to pulley

$= mR^2 + \frac{MR^2}{2}$

$= 15\pi^2$

$\omega_n = \sqrt{\frac{K\pi^2}{15\pi^2}} = \sqrt{\frac{1500}{15}} \Rightarrow \boxed{\omega_n = 10 \text{ rad/s}}$

Shortcut method

$\omega_n = \sqrt{\frac{K(\text{distance from pivot point})^2}{m(\text{dist}^n \text{ from pivot point})^2}}$

or $\omega_n = \sqrt{\frac{K(\text{dist}^n \text{ from pivot pt})^2}{\text{Inertia of system}}}$

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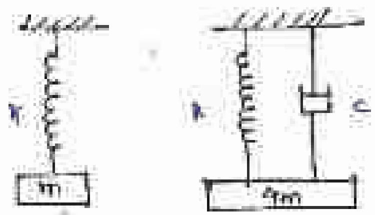

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$\omega_d = 20$ rad/s

$\xi = (?)$

$\omega_n = \sqrt{\frac{K}{m}} \Rightarrow \omega_n \propto \frac{1}{\sqrt{m}}$

$\frac{\omega_{n_1}}{\omega_{n_2}} = \sqrt{\frac{m_2}{m_1}} \Rightarrow \frac{10}{\omega_{n_2}} = \sqrt{\frac{4m}{m}}$

$\boxed{\omega_{n_2} = 25 \text{ rad/s}}$

$\rightarrow \omega_d = \omega_{n_2}\sqrt{1-\xi^2}$

$20 = 25\sqrt{1-\xi^2}$

$\left(\frac{4}{5}\right)^2 = 1-\xi^2 \Rightarrow \xi^2 = \frac{9}{25} \Rightarrow \boxed{\xi = 3/5} = 0.6 = 60\%$

20

$M = 240$ kg

$K_{eq} = k_1 + k_2 + k_3 + k_4$

$= 16 + 16 + 32 + 32$

$= 96$ kN/m

Resonance

$\omega = \omega_n \Rightarrow \omega_n = \sqrt{\frac{K_{eq}}{M}} = \sqrt{\frac{96\times10^6}{240}}$

$\boxed{\omega_n = 632.5 \text{ rad/s}}$

$\frac{2\pi N}{60} = 632.5$

$\boxed{N = 6040 \text{ rpm}}$

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$\Rightarrow I_p\ddot{\theta} + F_s(l\cos\theta) + F_d(2l\cos\theta) = 0$

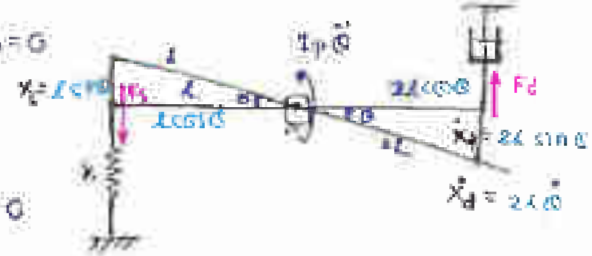
$\Rightarrow I_p\ddot{\theta} + F_s(l) + F_d(2l) = 0$

$\Rightarrow I_p\ddot{\theta} + Kx_s(l) + C\dot{x}_d(2l) = 0$

$\Rightarrow I_p\ddot{\theta} + K(l\theta)(l) + C(2l\dot{\theta})(2l) = 0$

$\Rightarrow I_p\ddot{\theta} + Kl^2\theta + C(4l^2)\dot{\theta} = 0$

$\Rightarrow I_p\ddot{\theta} + 4l^2C\dot{\theta} + Kl^2\theta = 0$



comparing $I_{eq}\ddot{\theta} + C_{eq}\dot{\theta} + K_{eq}\theta = 0$

$I_{eq} = I_p$ | $\boxed{C_{eq} = 4l^2C}$ | $\boxed{K_{eq} = Kl^2}$

$I_{CG} = \frac{m(3l)^2}{12}$

$I_p = I_{CG} + m(l/2)^2$

$= \frac{9ml^2}{12} + \frac{ml^2}{4}$

$\boxed{I_p = ml^2}$

$\xi = \frac{C}{C_c}$

$= \frac{C_{eq}}{2m\omega_n}$ (in translation) $= \frac{C_{eq}}{2I_{eq}\omega_n}$ (in rotation)

$C \rightarrow C_{eq}$, $m \rightarrow I$

$= \frac{C_{eq}}{2\sqrt{K_{eq}m}} = \frac{C_{eq}}{2\sqrt{I_{eq}K_{eq}}}$ (in rotation)

$= \frac{4l^2C}{2\sqrt{ml^2(Kl^2)}} = \frac{4C}{2\sqrt{Km}}$

$\boxed{\xi = \frac{2C}{\sqrt{Km}}}$

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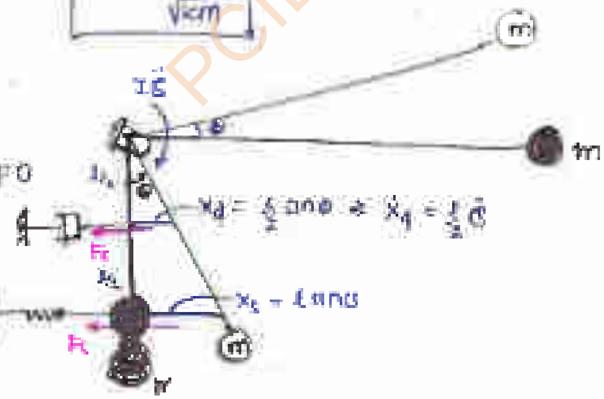


$\Rightarrow I_p\ddot{\theta} + F_d\left(\frac{l}{2}\cos\theta\right) + F_s(l\cos\theta) = 0$

$\Rightarrow I_p\ddot{\theta} + C\left(\frac{l}{2}\dot{\theta}\right)\frac{l}{2} + K(l\theta)l = 0$

$\Rightarrow I_p\ddot{\theta} + \frac{Cl^2}{4}\dot{\theta} + Kl^2\theta = 0$

compare

$I_p\ddot{\theta} + C_{eq}\dot{\theta} + K_{eq}\theta = 0$

$I_{eq} = I_p$

$I_{eq} = ml^2 + m(2l)^2$

$\boxed{I_{eq} = 5ml^2}$

| $C_{eq} = \frac{Cl^2}{4}$ | $K_{eq} = Kl^2$

$$= \sqrt{\frac{K_{eq}}{I_{eq}}} = \sqrt{\frac{K\ell^2}{5m\ell^2}} = \sqrt{\frac{K}{5m}}$$

$$= \sqrt{\frac{400}{5(10)}}$$

$$\boxed{\omega_n = 2.83 \text{ rad/s}}$$

→ ξ = (3)

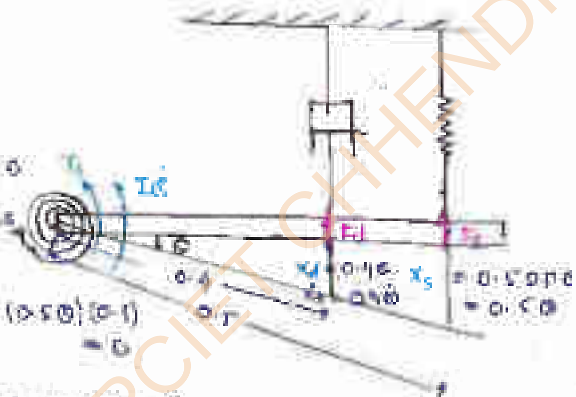
$$\xi = \frac{c}{c_c} = \frac{c}{2m\omega_n} \quad \text{(or)} \quad \frac{c}{2\sqrt{km}} \quad \text{(in translation)}$$

$$\xi = \frac{C_{eq}}{2 I_{eq}\omega_n} \quad \text{(or)} \quad \frac{C_{eq}}{2\sqrt{K_{eq} I_{eq}}} \quad \text{(in rotation)}$$

$$\xi = \frac{c\ell^2/4}{2\sqrt{K\ell^2 \cdot 5m\ell^2}} = \frac{c}{8\sqrt{5Km}} = \frac{400}{8\sqrt{5\times 400\times 10}}$$

$$\boxed{\xi = 0.35}$$

(c)



$$\Rightarrow I\ddot\theta + T_t + F_d(0.4\cos\theta) + F_s(0.5\cos\theta) = 0$$

$$\Rightarrow I\ddot\theta + T_t + F_d(0.4) + F_s(0.5) = 0$$

$$\Rightarrow I\ddot\theta + T_t + c\dot x_d(0.4) + Kx_s(0.5) = 0$$

$$\Rightarrow I\ddot\theta + T_t + c(0.4\dot\theta)(0.4) + K(0.5\theta)(0.5) = 0$$

$$\Rightarrow I\ddot\theta + K_t\theta + 0.16c\dot\theta + 0.25K\theta = 0$$

$$\Rightarrow I\ddot\theta + 0.16c\dot\theta + (K_t + 0.25K)\theta = 0$$

$I_p = I_{eq}$

$I_{cm} = \frac{m\ell^2}{12}$

$I_{xx'} = \frac{m\ell^2}{9}$

$I_p = \frac{m\ell^2}{3} = 0.433$ kg·m²

$C_{eq} = 0.16\,C$

$C_{eq} = 0.16(500)$

$C_{eq} = 80$ $\frac{N\text{-}m\text{-}s}{rad}$ ← for torsional

$c = \frac{\text{Torque}}{\dot\theta}$

$K_{eq} = k_t + 0.25K$

$K_{eq} = 1 + (0.25)(2)$

$= 1.5$ $\frac{kN\text{-}m}{rad}$ ← for rotation

$K = \frac{\text{Torque}}{\theta}$

$\omega_n = \sqrt{\frac{K_t}{I_m}} = \sqrt{\frac{[illegible]}{0.533}}$

$\boxed{\omega_n = 42.43 \text{ rad/s}}$

[Q8] $\frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \omega_n^2 x = 0$

$\rightarrow x = X e^{-\xi\omega_n t} \sin(\omega_d t + \phi) \Rightarrow x(t) = X e^{-\xi\omega_n t}$

$\rightarrow x(nT_d) = X e^{-\xi\omega_n n \left(\frac{2\pi}{\omega_n\sqrt{1-\xi^2}}\right)}$

$\left\{ t = nT_d = n\frac{2\pi}{\omega_d} = \frac{n 2\pi}{\omega_n\sqrt{1-\xi^2}} \right.$

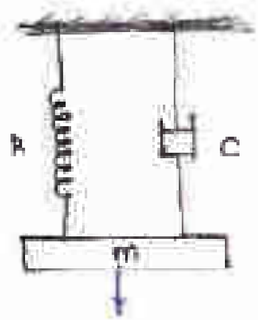
$\boxed{x(nT_d) = X e^{-2n\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}}$

[Q9] $\xi = \frac{c}{2\sqrt{km}} = \frac{25}{2\sqrt{100\times1}}$

$= \frac{25}{20} \Rightarrow \boxed{\xi = 1.25}$

$\rightarrow$ **Forced Vibration**:

- (i) const. forcing — $\boxed{F = F_0}$
- (ii) Harmonic forcing — $\boxed{F = F_0 \sin\omega t}$
- (iii) Random
- (iv) Chaos



$F = F_0 \sin\omega t$

$m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$

$\Rightarrow \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\sin\omega t$

Sol^n $x = f(t)$

$x = C.F + P.I$

C.F : $\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$

$\Rightarrow D^2x + \frac{c}{m}Dx + \frac{k}{m}x = 0$

$\Rightarrow \left[D^2 + \frac{c}{m}D + \frac{k}{m}\right]x = 0$

$\boxed{C.F = X e^{-\xi\omega_n t} \sin(\omega_d t + \phi)}$

$$\frac{\ldots}{D^2 + \frac{c}{m}D + \frac{k}{m}}$$

{ $D^2 = -\omega^2$

$$= \frac{F_0/m \; \sin\omega t}{-\omega^2 + \frac{c}{m}D + \omega_n^2}$$

$$= \frac{(F_0/m)\; \sin\omega t}{(\omega_n^2 - \omega^2) + \frac{c}{m}D} \times \frac{(\omega_n^2 - \omega^2) - c/m\, D}{(\omega_n^2 - \omega^2) - c/m\, D}$$

$$= \frac{(F_0/m)\left[(\omega_n^2 - \omega^2)\sin\omega t - c/m\, D \sin\omega t\right]}{(\omega_n^2 - \omega^2)^2 - \left(\frac{c}{m}\right)^2 D^2}$$

{ $D = \frac{d}{dt}$

$$= \frac{(F_0/m)\left[(\omega_n^2 - \omega^2)\sin\omega t - \frac{c\omega}{m}\cos\omega t\right]}{(\omega_n^2 - \omega^2)^2 - \left(\frac{c}{m}\right)^2(-\omega^2)}$$

$$= \frac{(F_0/m)\left[(\omega_n^2 - \omega^2)\sin\omega t - \frac{c\omega}{m}\cos\omega t\right]}{(\omega_n^2 - \omega^2)^2 + \left(\frac{c\omega}{m}\right)^2}$$

Let: $\omega_n^2 - \omega^2 = R\cos\phi$

$\frac{c\omega}{m} = R\sin\phi \Rightarrow R^2\cos^2\phi + R^2\sin^2\phi = (\omega_n^2 - \omega^2)^2 + \left(\frac{c\omega}{m}\right)^2$

$$\boxed{R^2 = (\omega_n^2 - \omega^2)^2 + \left(\frac{c\omega}{m}\right)^2}$$

$$P.I. = \frac{F_0/m\left[R\cos\phi\sin\omega t - R\sin\phi\cos\omega t\right]}{R^2}$$

$$= \frac{F_0/m\; \sin(\omega t - \phi)}{R}$$

$$\boxed{P.I. = \frac{F_0/m\; \sin(\omega t - \phi)}{\sqrt{(\omega_n^2 - \omega^2)^2 + \left(\frac{c\omega}{m}\right)^2}}}$$

$$\Rightarrow P.I. = \frac{F_0/m\; \sin(\omega t - \phi)}{\sqrt{\omega_n^4\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{c\,\omega\,\omega_n^2}{m\,\omega_n^2}\right)^2}} = \frac{F_0/m\; \sin(\omega t - \phi)}{\omega_n^2\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{c}{m\omega_n}\cdot\frac{\omega}{\omega_n}\right)^2}}$$

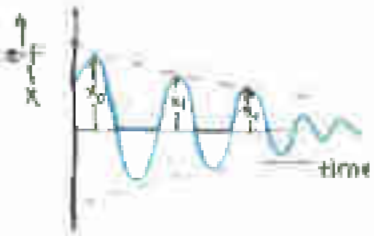
$$P.I. = \frac{F_0/m\; \sin(\omega t - \phi)}{k/m\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$

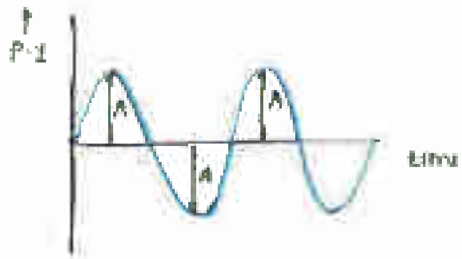
$$\boxed{P.I. = \frac{F_0/k\; \sin(\omega t - \phi)}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}}$$

$$x = X e^{-\xi \omega_n t} \sin(\omega_d t + \phi) + \frac{F_0/K \; \sin(\omega t - \phi)}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi \frac{\omega}{\omega_n}\right]^2}}$$

Amplitude $= X e^{-\xi \omega_n t}$

Steady state response $= \dfrac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi \frac{\omega}{\omega_n}\right]^2}}$





total solⁿ

$x = CF + PF$

⇒ Steady state Response (or) Dynamic Amplitude (A)

– The amplitude of P.I is always constant with respect to time therefore it is known steady state response.

$$A = \frac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi \frac{\omega}{\omega_n}\right]^2}}$$

[3] Assume $\xi = 0.1$

A_1 amplitude @ resonance

$\frac{\omega}{\omega_n} = 1 \Rightarrow$ $\boxed{\omega = \omega_n}$ $\boxed{r_1 = 1}$

at $\frac{\omega}{\omega_n} = 0.5 \Rightarrow (r)$ $\boxed{r_2 = 0.5}$

$$A_1 = \frac{F_0/K}{\sqrt{(1-r_1^2)^2 + (2\xi r_1)^2}} = \frac{F_0/K}{2\xi} = 10 \text{ cm}$$

$$A_2 = \frac{F_0/K}{\sqrt{(1-0.25)^2 + (\xi)^2}} = \frac{F_0/K}{\sqrt{0.5625 + \xi^2}} = X$$

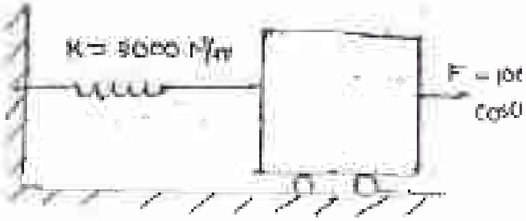
$(2\xi)(10) = X\sqrt{0.5625 + \xi^2} \Rightarrow 2 = X(0.7566)$

$X = 2.64$ cm

$m = 40$ mm (?)
$K = 3000$ N/m

$F(t) = 100 \cos(100t)$
$= F_0 \cos(\omega t)$

$F_0 = 100$, $\omega = 100$



— dashpot is absent

$\boxed{\xi = 0}$

$$A = \frac{F_0/K}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{100/3000}{(1-r^2)} = 0.05$$

$\boxed{r = 0.577} = \omega/\omega_n$

$\rightarrow \frac{\omega}{\omega_n} = 100/0.577$

$\boxed{\omega_n = 173.20 \text{ rad/s}}$ $\rightarrow$ $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3000}{m}} = 173.20$

$\boxed{m = 0.1 \text{ kg}}$

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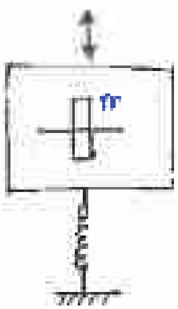


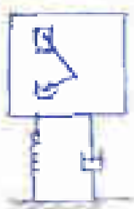
$M = 100$ kg ✓
$m = 20$ kg
$e = 0.5$ mm
$K_{eq} = 85$ kN/m
$\omega = 20\pi$
$\xi = 0 \rightarrow$ Because damping is negligible

⇒ Rotating unbalance

$F_0 = me\omega^2$

⇒ Reciprocating unbalance

Total body or foundation mass M is responsible to take in only that



$F_1 = mr\omega^2 \cos\theta$
$F = F_0 \cos\omega t$
$\boxed{F_0 = mr\omega^2}$

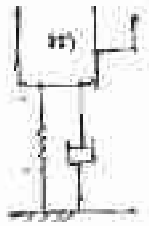
$$A = \frac{F_0/K}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{F_0}{K(1-r^2)}$$

$\left\{ r = \frac{\omega}{\omega_n} = \frac{20\pi}{\sqrt{\frac{K}{100}}} \right.$

now $r = \frac{\omega}{\omega_n} = \frac{\omega}{\sqrt{\frac{K}{M}}} = \frac{20\pi}{\sqrt{\frac{85000}{100}}} = 2.155$

∴ $F_0 = me\omega^2 = 39.47$ $\rightarrow$ $\boxed{A = -1.279 \times 10^{-4} \text{ m}}$

Q5



$m = 10\ kg$

$k = 6250\ N/m$

$F = F_0 \cos \omega t$

$F_0 = 10,\ \omega = 25$

$A = 40\ mm = 0.04\ m$

$c = ?$

$$r = \frac{\omega}{\omega_n} = \frac{25}{\sqrt{\frac{6250}{10}}} = 1 \rightarrow \boxed{r = 1}$$

Resonance

$$A = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{10}{6250\,(2 \times \xi \times 1)} = 0.04$$

$$\boxed{\xi = 0.02}$$

$$\xi = \frac{c}{2m\omega_n} \rightarrow 0.02 = \frac{c}{2(10)\sqrt{\frac{6250}{10}}}$$

$$\boxed{c = 10\ Ns/m}$$

Q3

$F_0 = 10\ N$

$k = 100\ N/m$

$\xi = 0.2$

$\omega_n = 10\,\omega$

$x = ?$

$$\frac{\omega}{\omega_n} = r = 0.1$$

$$x = \frac{\frac{F_0}{k}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{10}{100\sqrt{(1-0.1^2)^2 + (2 \times 0.2 \times 0.1)^2}}$$

$$\boxed{x = 0.067\ m} \approx 0.07\ m$$

Q5

$F_0 = 100\ N$

$\xi = 0.25$

$k = 10000\ N/m$

→ Resonance $\omega = \omega_n$

$$\boxed{r = 1}$$

$$x = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$= \frac{100}{10000\,(2 \times 0.25 \times 1)}$$

$$= 0.02\ m$$

$$\boxed{x = 20\ mm}$$

Steady state Response $A = \dfrac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

$\rightarrow F = F_0 \cos \omega t$

if $\omega = 0 \Rightarrow F = F_0$

Static deflection $\delta = \dfrac{F_0}{k}$

Magnification factor = $\dfrac{\text{Dynamic Amplitude}}{\text{static deflection}}$

$$\boxed{M.F. = \frac{A}{F_0/k}}$$

$$\Rightarrow \boxed{M.F = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}}$$

$M.F = f(r) = f(\xi, r)$

if ξ is constant

$MF = f(r)$

$\rightarrow$ for max^n or min^n

$\frac{d}{dr}(MF) = 0$

$\Rightarrow \frac{d}{dr}\left[\frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}\right] = 0$

$\Rightarrow \frac{d}{dr}\left[\left((1-r^2)^2 + (2\xi r)^2\right)^{-1/2}\right] = 0$

$-\frac{1}{2}\left[(1-r^2)^2 + (2\xi r)^2\right]^{-3/2}\left[2(1-r^2)(0-2r) + 2(2\xi r)(2\xi)\right] = 0$

$4r(1-r^2) = 8\xi^2 r \rightarrow 1 - r^2 = 2\xi^2$

$\xi = \sqrt{\frac{1-r^2}{2}}$

$$\boxed{r_{opt} = \sqrt{1 - 2\xi^2}}$$

ξ	0	0.1	0.2	0.4	0.5
r_{opt}	1	0.989	0.959	0.824	0.707

$\sqrt{(1-r^2)^2 + (2\xi r)^2}$

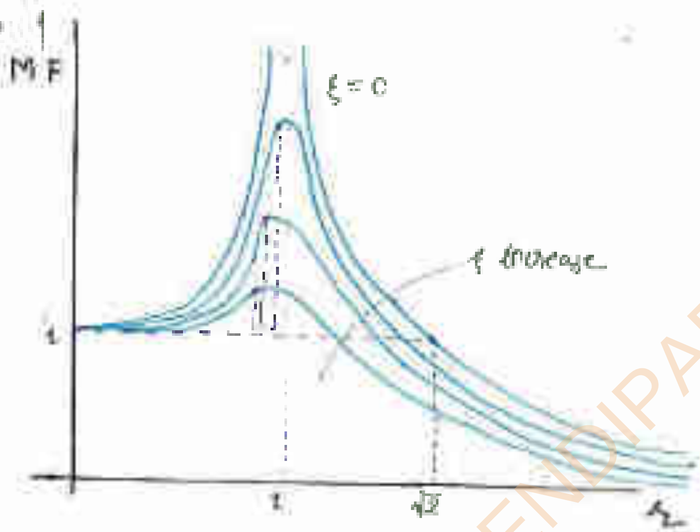
If $r = 0$, $\boxed{MF = 1}$

$r = 1$ @ Resonance, $MF = \frac{1}{2\xi}$

@ $r = 1$, $\xi = 0$, $\boxed{MF = \infty}$

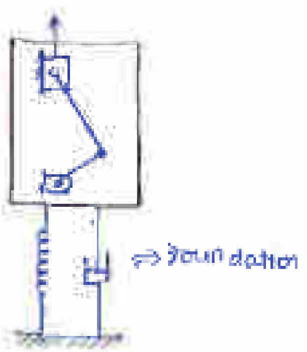
If $\xi = 0$, $MF = \pm \frac{1}{1 - r^2}$

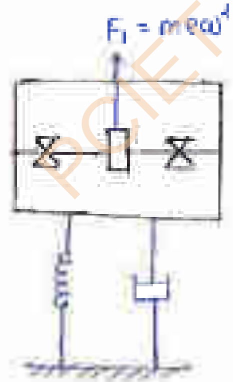
ξ increases → r slightly increase

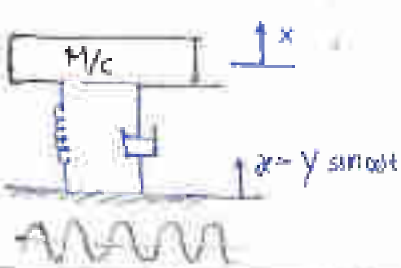


⇒ Vibration Isolation:

$F_t = m e \omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$







Force transmissibility — Motion transmissibility

$\rightarrow \quad m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t$

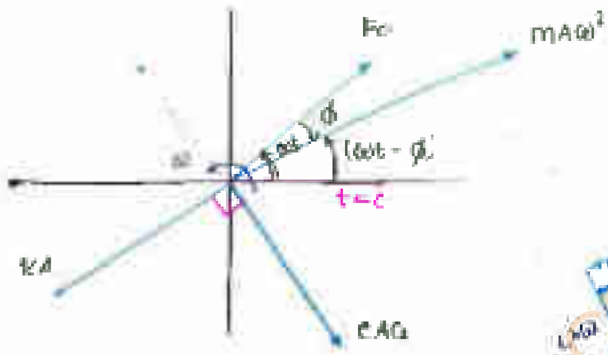
soln $\Rightarrow \quad x = A\sin(\omega t - \phi)$

$\dot{x} = A\omega\cos(\omega t - \phi) = A\omega\sin[(\omega t - \phi) + \pi/2]$

$\ddot{x} = -A\omega^2\sin(\omega t - \phi)$

$\Rightarrow -mA\omega^2\sin(\omega t - \phi) + cA\omega\sin[(\omega t - \phi) + \pi/2] + kA\sin(\omega t - \phi) = F_0\sin\omega t$

$\Rightarrow F_0\sin(\omega t) + mA\omega^2\sin(\omega t - \phi) - cA\omega\sin[(\omega t - \phi) + \pi/2] - kA\sin(\omega t - \phi) = 0$



$\tan\phi = \dfrac{cA\omega}{KA - mA\omega^2}$

$\rightarrow \tan\phi = \dfrac{c\omega}{k - m\omega^2}$

$\Rightarrow \tan\phi = \dfrac{\frac{c\omega}{m}}{\frac{k}{m} - \omega^2} = \dfrac{\frac{c\omega}{m}}{\omega_n^2 - \omega^2} = \dfrac{\frac{c\omega}{m\omega_n^2}}{\frac{\omega_n^2}{\omega_n^2} - \frac{\omega^2}{\omega_n^2}}$

$= \dfrac{\frac{2c\omega}{2m\omega_n} \cdot \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$

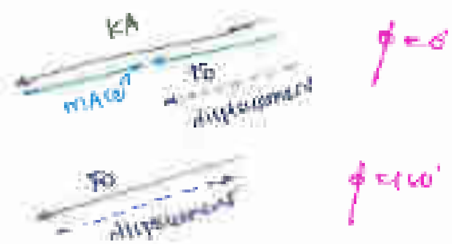
$$\boxed{\tan\phi = \frac{2\xi r}{1 - r^2}}$$

ϕ = phase lag betn displacement vector and F_0

$\xi = 0 \Rightarrow$ undamped

$\tan\phi = 0$

$\boxed{\phi = 0° \ \& \ 180°}$



→ If $\phi \to 0°$ the External force & displacement are in same direction, if $\phi \to 180°$ the displacement will be opposite to the direction applied force.

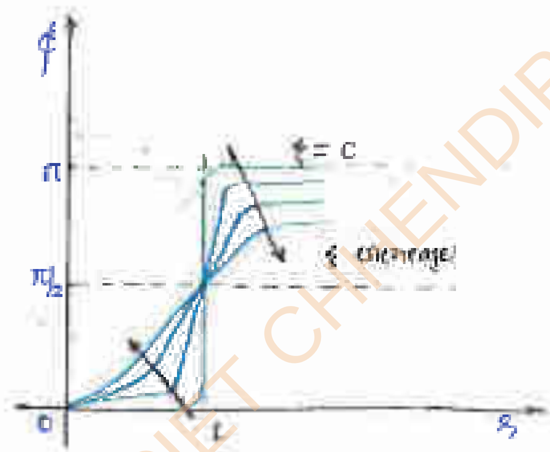
(a) $\xi = 1$

$\tan\phi = \infty$

$\boxed{\phi = \pi/2}$

→ GATE-Q:

- when ϕ tends to 0° angle the Fo & displacement are in same direction
- If ϕ tends to 180° then Fo & displacement in opposite dirⁿ



⇒ Transmission Ratio or Transmissibility (ε)
(T·R)

a] Force Transmissibility

$\epsilon = \frac{O/P}{I/P} = \frac{\text{effect}}{\text{cause}}$

ϵ = Force transmitted to foundation / Impressing Disturbing force

$$\boxed{\epsilon = \frac{F_T}{F_0}}$$

→ $F_T = \sqrt{(F_s)^2 + (F_d)^2} = \sqrt{(KA)^2 + (c\omega A)^2}$

$$\boxed{F_T = A\sqrt{K^2 + c^2\omega^2}}$$

$$= \frac{F_0}{K\sqrt{(1-r^2)^2+(2\xi r)^2}} \times \frac{\sqrt{K^2+(c\omega)^2}}{F_0}$$

$$= \frac{\sqrt{\frac{K^2}{K^2}+\left(\frac{c\omega}{K}\right)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$= \frac{\sqrt{1+\left(\frac{c\omega}{K}\cdot\frac{\omega_n^2}{\omega_n^2}\right)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\epsilon = \frac{\sqrt{1+\left(\frac{c\,\omega}{K\cdot\omega_n\,\omega_n}\cdot\frac{K}{m}\right)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$= \frac{\sqrt{1+\left(\frac{c}{m\omega_n}\cdot r\right)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\boxed{\epsilon = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}}$$

Special case

i) if $r = 0 \Rightarrow \boxed{\epsilon = 1}$

imp ii) $\xi = 0 \rightarrow \boxed{\epsilon = \pm\frac{1}{1-r^2}}$

iii) $r = 1 \Rightarrow \omega = \omega_n \Rightarrow$ Resonance

$$\epsilon = \frac{1+(2\xi)^2}{\sqrt{(1-1)^2+(2\xi)^2}} = \frac{\sqrt{1+(2\xi)^2}}{2\xi}$$

$$\boxed{\epsilon = \frac{\sqrt{1+4\xi^2}}{2\xi}}$$

iv) $r = 1$, $\xi = 0$

$\boxed{\epsilon = \infty}$

v) $r = \sqrt{2} \rightarrow \epsilon = \frac{\sqrt{1+8\xi^2}}{\sqrt{1+8\xi^2}} \Rightarrow \boxed{\epsilon = 1}$ {at $r = \sqrt{2}$ ... $\Rightarrow \boxed{\epsilon = 1}$

→ (i) $0 < r < \sqrt{2}$

$\epsilon > 1$

$\frac{F_T}{F_0} > 1$

$F_T > F_0$

$\xi \uparrow$, $\epsilon \downarrow$ } damping helpful, spring creating problem

→ (ii) $r = \sqrt{2}$

$\epsilon = 1$

$F_T = F_0$

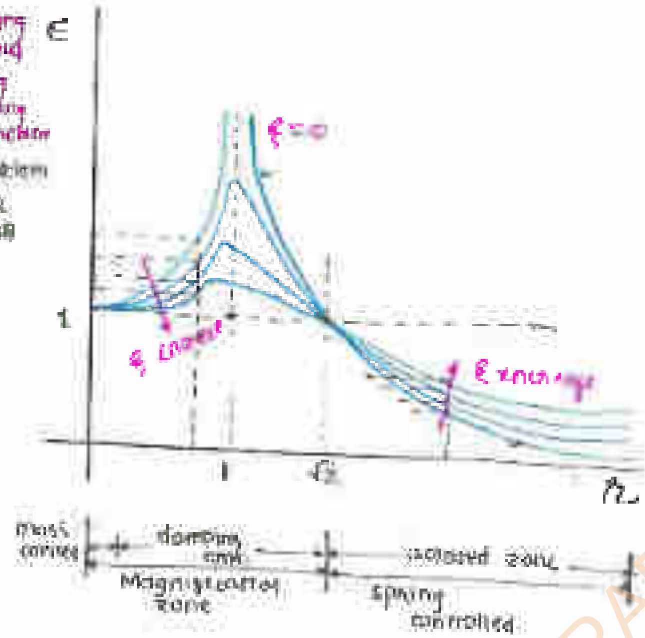
→ (iii) $r > \sqrt{2}$

$\epsilon < 1$

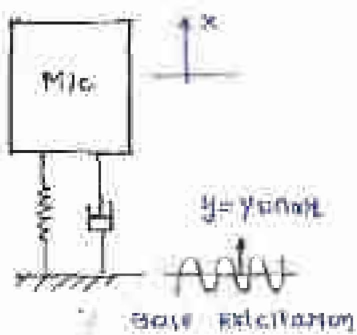
$F_T < F_0$

$\xi \uparrow$, $\epsilon \uparrow$

damping creating problem, springs are helpful



⇒ Motion Transmissibility :



$\epsilon = \frac{\text{amplitude of mass } (x)}{\text{amplitude of foundation } (y)}$

$y = Y \sin \omega t$

$\dot{y} = Y\omega \cos \omega t$

$\Rightarrow m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$

$\Rightarrow m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

$\Rightarrow m\ddot{x} + c\dot{x} + kx = cY\omega \cos \omega t + kY \sin \omega t$

$cY\omega = R \sin \alpha$

$kY = R \cos \alpha$

$\Rightarrow m\ddot{x} + c\dot{x} + kx = R \sin(\omega t + \alpha)$

$= F_0 \sin \omega_1$

$$\epsilon = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$

$m = 1$ kg

$x_1 = x_0/2$

$\delta = \frac{1}{n} \ln\left(\frac{x_0}{x_1}\right)$

$\boxed{\delta = 0.6931} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.6931$

$9.06\xi = \sqrt{1-\xi^2}$

$82.08\,\xi^2 = 1 - \xi^2$

$\boxed{\xi = 0.1097}$

$\rightarrow \xi = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}}$

$\boxed{c = 2.19 \ \text{N·s/m}}$

CRQ. 81 | Static deflection $= F_0/k = 3$ mm

$\omega = 20$ rad/s $\rightarrow \boxed{r = 2}$

$\omega_n = \sqrt{\frac{k}{m}} = 10$ rad/s $\Rightarrow \omega_n = 10$ rad/s

$r = \omega/\omega_n = 2$

$A = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

$A = 0.999 \approx 1$ mm

$\rightarrow r > 1 \Rightarrow$ for displacement in opposite dirn

$\tan\phi = \frac{2\xi r}{1-r^2} \Rightarrow \phi = -8.46°$

$\phi = 171.54° \approx 180 \Rightarrow$ (opposite)

95 | M.F $= 20 \Rightarrow$ @ $r = 1$ (resonance)

M.F $= \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

$20 = \frac{1}{\sqrt{(2\xi)^2}} = \frac{1}{2\xi}$

$\boxed{\xi = 0.025}$

$X = 0.01$ mm

$M = 50$ kg

$y(t) = 0.2 \sin(200\pi t)$

$Y = 0.2$

$\omega = 200\pi$ rad/s

No damping $\rightarrow \xi = 0$

$\rightarrow \quad \epsilon = \dfrac{X}{Y} = \dfrac{0.01}{0.2}$

$\boxed{\epsilon = 0.05}$

$\rightarrow$ Transmissibility $= \dfrac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \pm \dfrac{1}{1-r^2} = \epsilon$

$0.05 = \pm \dfrac{1}{1-r^2}$

$1 - r^2 = \pm 20$

$r^2 = 21 \Rightarrow \boxed{r = 4.58}$

$\rightarrow \quad \dfrac{\omega}{\omega_n} = 4.58 \Rightarrow \boxed{\omega_n = 137.04} \text{ rad/s}$

$\sqrt{\dfrac{k}{m}} = 137.04$

$\boxed{k = 939.04 \text{ kN/m}}$

Q8

$5\ddot{x} + 20\dot{x} + 80x = 5\cos 4t$

$m\ddot{x} + c\dot{x} + kx = F_0 \sin(\omega t + \phi)$

$= F_0 \cos \omega t$

$m = 5$

$c = 20$

$k = 80$

$F_0 = 5$

$\omega = 4$

ii) $\xi = \dfrac{c}{c_c} = \dfrac{c}{2\sqrt{km}}$

$= \dfrac{20}{2\sqrt{5 \times 80}}$

$\boxed{\xi = 0.5}$

i) $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{80}{5}} = \sqrt{16} \Rightarrow \boxed{\omega_n = 4}$

4) $M.F = \dfrac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \dfrac{1}{2 \times 0.5 \times 1}$

$\boxed{M.F = 1}$

b) $A = \dfrac{F_0/k}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

@ $r = \omega = \omega_n = 1$

$= \dfrac{F_0}{k \times 2\xi} = \dfrac{8}{40 \times 2 \times 0.2}$

$\boxed{A = 0.1}$

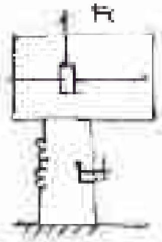
CRO 39] $M = 250$ kg

$k_{eq} = 100$ kN/m

$F_0 = 350$ N

$N = 3600$ rpm

$\xi = 0.15$



$\rightarrow \omega = \dfrac{2\pi N}{60} = 376.8$ rad/s

$\rightarrow \omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{100 \times 10^3}{250}} = 20$

$r = \dfrac{\omega}{\omega_n} = \dfrac{376.8}{20} \Rightarrow \boxed{r = 18.84}$

$\epsilon = \dfrac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} = \dfrac{\sqrt{1+(2\times0.15\times18.84)^2}}{\sqrt{(1-18.84^2)^2+(2\times0.15\times18.84)^2}}$

$= \dfrac{5.7397}{352.94}$

$\boxed{\epsilon = 0.0162}$

CRO 44] $m = 1$ kg {Base excitation}

$\omega = 2\pi \times 60$

$\epsilon = 0.05$

natural frequency significantly less than 60 Hz

$\Rightarrow \xi$ neglected

$\epsilon = \dfrac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$

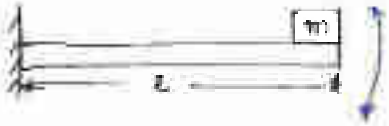
$0.05 = \dfrac{1}{1-r^2} \rightarrow \boxed{r = 4.58}$

$\dfrac{\omega}{\omega_n} = 4.58 \Rightarrow \boxed{\omega_n = 82.2 \text{ rad/s}}$

$82.2 = \sqrt{\dfrac{k}{m}}$

$\boxed{k = 6760.8 \text{ N/m}}$

3) Vibration of beam due to concentrated mass



$$\omega_n = \sqrt{\frac{g}{\delta_{max}}}$$

ω_n doesn't depend on the g
→ if g changes, δ also changes

$$\omega_n = \sqrt{\frac{K}{m}} \qquad \text{where } K = \frac{3EI}{l^3}$$

$$y = \frac{wl^3}{3EI} \Rightarrow \boxed{K = \frac{3EI}{l^3}}$$

Theorem of Moment area (?)

$$\rightarrow \omega_n = \sqrt{\frac{g}{\frac{mgl^3}{3EI}}} \Rightarrow \sqrt{\frac{3EI}{ml^3}}$$

$$\boxed{\omega_n = \sqrt{\frac{3EI}{ml^3}}}$$

Deflection

- Calculus
  - Double integration
  - Macaulay
- Geometry
  - Conjugate beam
  - Moment area
- Energy
  - Strain energy
  - Castigliano's theorem

Q. 19)
EI = const.
l = 0.01 m
m = 0.05 kg
f_n = 100 Hz

$$\rightarrow \omega_n = \sqrt{\frac{3EI}{ml^3}} \Rightarrow 2\pi(100) = \sqrt{\frac{3EI}{(0.1)(0.05)^3}}$$

$$\boxed{EI = 0.065 \ N m^2}$$

$l = 1$ m

$E_s = 200$ GPa

critically damped $\boxed{\xi = 1}$

$$\omega_n = \sqrt{\frac{3EI}{m\,l^3}} = \sqrt{\frac{3 \times 200 \times 10^3 \times \frac{25^4}{12}}{(20)(1000)^3} \times \frac{1}{(10^{-3})}} \qquad \frac{\frac{N}{mm^2} \cdot mm^4}{mm^3} = \frac{N}{mm}$$

ω_n = ~~[illegible]~~ rad/s

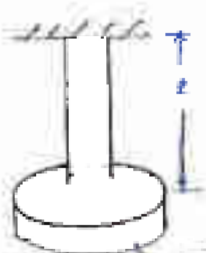
= 31.250 rad/s

$c_c = 2\omega_n m = 2(20)(31.250)$

$\boxed{c_c = 1250 \text{ Ns/m}}$

⇒ **Torsional vibration**

1)



mass moment of inertia of rotor

In straight of the area No.1 In dynamics the mass M.M.I

$\to I\ddot{\theta} + q\theta = 0$

$\ddot{\theta} + \frac{q}{I}\theta = 0$

$$\boxed{\omega_n = \sqrt{\frac{q}{I}}}$$

where $q = \frac{GJ}{l}$

where J = polar area MOI (shaft)

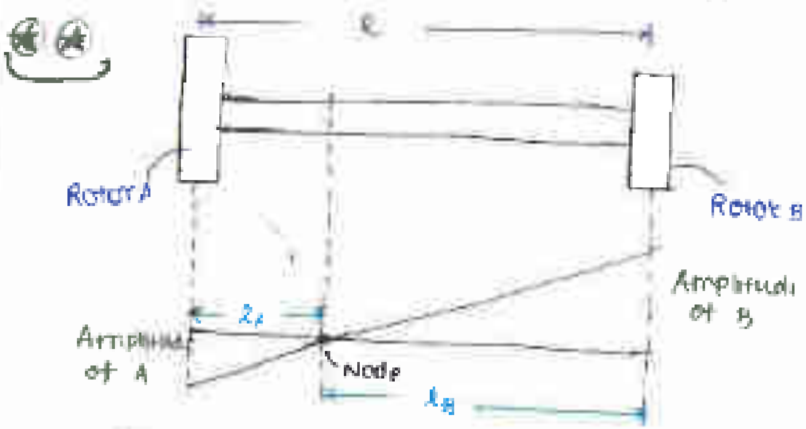
→ If mass MOI of shaft is also considered

$$\boxed{\omega_n = \sqrt{\frac{q}{I_{rotor} + \frac{I_{shaft}}{3}}}}$$

2) When rotor are moving (Amplitude) in same direction

→ The natural frequency of system due to movement of rotor will be zero. & system will not vibrate at all

Case-(ii): When rotors are moving in opposite direction



→ $(\omega_n)_A = (\omega_n)_B$

$\left(\sqrt{\frac{q}{I}}\right)_A = \left(\sqrt{\frac{q}{I}}\right)_B$ { torsional rigidity GJ = const, $q = \frac{GJ}{l}$ }

$\left(\frac{GJ}{lI}\right)_A = \left(\frac{GJ}{lI}\right)_B$

$$\boxed{l_A I_A = l_B I_B}$$

→ $\frac{l_A}{l_B} = \frac{I_B}{I_A}$

$$\boxed{l \propto \frac{1}{\text{Mass MOI of rotor}}}$$

→ $$\boxed{\frac{\text{Amplitude of rotor B}}{\text{Amplitude of rotor A}} = \frac{l_B}{l_A} = \frac{I_A}{I_B}}$$

If $I_A > I_B$ → $\frac{l_A}{l_B} < 1$ → $l_A < l_B$

→ The point on shaft where angular displacement is zero is known as Node.

→ The Node divides the length of shaft in inverse ratio of mass moment of inertia of the rotor attached at respective ends.

→ At Node the two shaft of different length l_A & l_B are clamped together makes an analogous system as a single shaft carrying two rotor at respective end.

→ Torsionally equivalent system

1) stepped shaft

$$\theta_{D/A} = \theta_{A/D}$$

$$\frac{T\, l_{eq}}{G\, J_{eq}} = \theta_{A/B} + \theta_{B/C} + \theta_{C/D}$$

$$= \frac{T l_1}{G J_1} + \frac{T l_2}{G J_2} + \frac{T l_3}{G J_3}$$

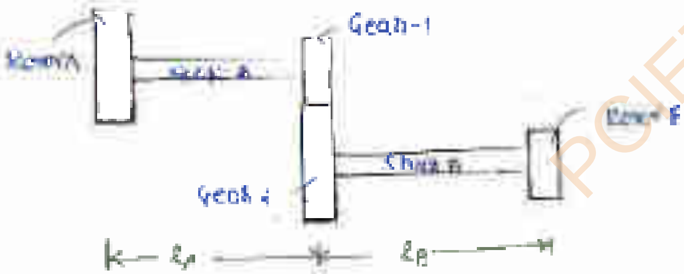
$$= \frac{l_1}{G J_1} + \frac{l_2}{G J_2} + \frac{l_3}{G J_3}$$

let $G_1 = G_2 = G_3 = G_{eq}$

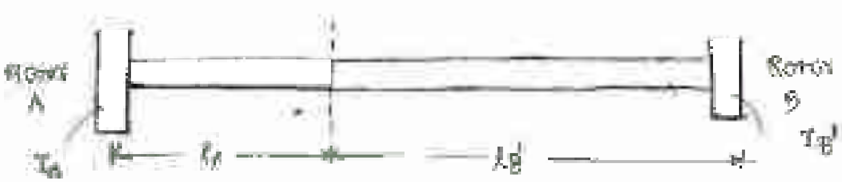
$$\frac{l_{eq}}{J_{eq}} = \frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}$$

$$\boxed{\frac{l_{eq}}{d_{eq}^4} = \frac{l_1}{d_1^4} + \frac{l_2}{d_2^4} + \frac{l_3}{d_3^4}}$$

2) geared system



- Backlash should not be present
- The kinetic energy & strain energy of real system & dynamically equivalent system should be same.
- The inertia of the shaft should be negligible



equating the P.E

$$\left(\frac{1}{2} T \theta\right)_{original} = \left(\frac{1}{2} T \theta\right)_{equi^n}$$

$$(T\theta)_{original} = (T\theta)_{equi^n}$$

$$\left(\frac{GJ}{\ell} \cdot \theta \cdot \theta\right)_{ori} = \left(\frac{GJ}{\ell} \cdot \theta \cdot \theta\right)_{eq^n}$$

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{\ell}$$

if $(GJ)_{orig} = (GJ)_{eq^n}$

$$\frac{\theta^2_{orig}}{\ell_B} = \frac{\theta^2_{eq}}{\ell_{eq}}$$

$$\ell_{eq} = \ell_B \left[\frac{\theta_B'}{\theta_B}\right]^2 \quad \text{— (i)}$$

total length of equivalent system $= \ell_A + \ell_B'$

where $\theta_{ori} = \theta_B$
$\theta_{equi} = \theta_B'$
$\ell_{eq} = \ell_B'$

⇒ equating the K.E

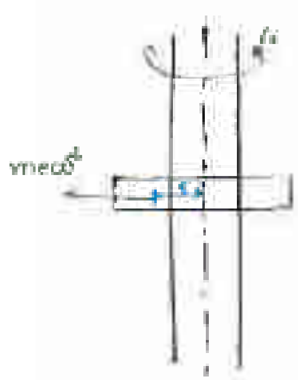
K.E of original system = K.E of equi. system

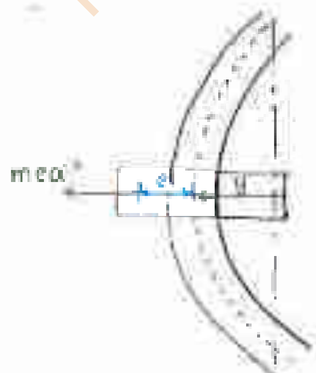
$$\Rightarrow \frac{1}{2} I_B \omega_B^2 = \frac{1}{2} I_B' (\omega_A)^2$$

$$\Rightarrow I_B \omega_B^2 = I_B' \omega_A^2$$

$$I_B' = I_B \left(\frac{\omega_B}{\omega_A}\right)^2$$

⇒ Critical / Whirling / Whipping / Balancing speed of shaft





where e = eccentricity of rotor
m = mass of rotor
k = stiffness of shaft

Centrifugal force = Restoring force

$\Rightarrow m(y+e)\omega^2 = Ky$

$\Rightarrow my\omega^2 + me\omega^2 = Ky$

$\Rightarrow my\omega^2 - Ky = -me\omega^2$

$\Rightarrow y(m\omega^2 - K) = -me\omega^2$

$\Rightarrow y = -\dfrac{me\omega^2}{m\omega^2 - K}$

$\Rightarrow y = \dfrac{e}{1 - \dfrac{K}{m\omega^2}}$

$$y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2}$$

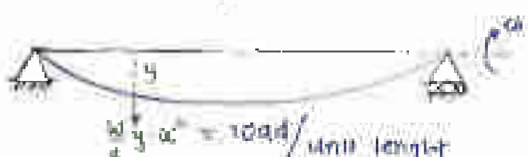
@ $\omega = \omega_n \rightarrow$ resonance $\boxed{y = \infty}$

⇒ When the speed of shaft becomes equal to natural frequency the deflection in shaft is infinite & the shaft vibrates violently & tends to fails.

→ Critical speed of the shaft is time dependent phenomenon. Hence in order to prevent failure of shaft we accelerate the shaft when it is about to send near to its critical speed.

Higher critical speeds

- Higher critical speeds of shaft is observed due to its own wt.
- The frequency will depend on the loading condition as well as end condition of shaft that is type of support beam provided.
- If a shaft is supported on short bearings it is analogous to simply supported & if it is supported on long bearing it is analogous to fixed end.



w = load/unit length

→ Shaft supported on short bearing

→ Let w is wt per unit length of shaft

$EI \frac{d^2y}{dx^2} = BM_{x-x}$

$\Rightarrow EI \frac{d^3y}{dx^3} = \frac{dBM_{x-x}}{dx}$

$\frac{dBM}{dx} = SF$

$\frac{dSF}{dx} = load$

$\Rightarrow$ $EI \frac{d^3y}{dx^3} = SF_{x-x}$

$\Rightarrow EI \frac{d^3y}{dx^3} = SF_{x-x} \Rightarrow EI \frac{d^4y}{dx^4} = \frac{dSF_{x-x}}{dx} = load/unit\ length$

$EI \frac{d^4y}{dx^4} = \frac{w}{g} y \omega^2$

$\Rightarrow \frac{d^4y}{dx^4} = \frac{w}{gEI} y \omega^2$ let $n^4 = \frac{w}{gEI} \omega^2$

$\Rightarrow \frac{d^4y}{dx^4} = n^4 y$

$\frac{d^4y}{dx^4} - n^4 y = 0$

$(D^4 - n^4) y = 0$

sol$^n \Rightarrow$ $y = f(x)$

sol^n = CF + PI

sol^n = CF

sol^n $D^4 - n^4 = 0$

$D^4 = n^4$

$D = \pm n, \pm in$

$\Rightarrow y = A \cos nx + B \sin nx + C \sinh(nx) + D \cosh(nx)$

$\rightarrow$ since shaft was simply supported (short bearing)

Boundary cond^n

@ $x = 0$; $y = 0$ — (i)

@ $x = L$; $y = 0$ — (ii)

@ $x = 0$; $\frac{d^2y}{dx^2} = 0$ — (iii)

@ $x = L$; $\frac{d^2y}{dx^2} = 0$ — (iv)

(i) $\rightarrow$ $0 = A + D$

(ii) $\rightarrow$ $0 = A \cos nL + B \sin nL + C \sinh(nL) + D \cosh(nL)$

(iii) $\rightarrow$ $\frac{dy}{dx} = -A(n) \sin nx + B(n) \cos nx + C(n) \cosh(nx) + D(n) \sinh(nx)$

$\frac{d^2y}{dx^2} = -An\cos\eta x - B\eta\sin\eta x + C\eta\sinh(\eta x) + D\eta\cosh(\eta x)$

(3) → $0 = -A\eta^2 + D\eta^2 \Rightarrow -A + D = 0$ — (5)

(4) → $0 = -A\eta^2\cos(\eta L) - B\eta^2\sin(\eta L) + C\eta^2\sinh(\eta L) + D\eta^2\cosh(\eta L)$

$B\sin(\eta L) + A\cos(\eta L) = C\sinh(\eta L) + D\cosh(\eta L)$

$A + D = 0$ — (1)

$-A + D = 0$ — (5)

$2D = 0$

$A = D$ $D = 0$

$B\sin(\eta L) = C\sinh(\eta L)$ — (5)

$B\sin(\eta L) + C\sin(h(\eta L))$ — (6)

$B\sin(\eta L) + C\sinh(\eta L) = 0$

$B\sin(\eta L) - C\sinh(\eta L) = 0$

$2B\sin(\eta L) = 0 \Rightarrow$ $\sin(\eta L) = 0$

so, $\eta L = \pi, 2\pi, 3\pi$

$\eta = \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L}$ ---

$\Rightarrow \left(\frac{\pi}{L}\right)^4 = \frac{\omega}{gEI}\omega_1^2$

$\omega_1 = \sqrt{\frac{gEI}{\omega}\cdot\left(\frac{\pi}{L}\right)^4}$

gravity = 1

$\omega_1 = \left(\frac{\pi}{L}\right)^2\sqrt{\frac{gEI}{W}}$ ← critical speed

→ Let $\eta^4 = \frac{\omega}{gEI}\omega^2$

$\left(\frac{2\pi}{L}\right)^4 = \frac{w}{gEI}\omega_2^2$

$\omega_2 = \sqrt{\frac{gEI}{w}\left(\frac{2\pi}{L}\right)^4}$

$\omega_2 = \left(\frac{2\pi}{L}\right)^2\sqrt{\frac{gEI}{w}}$

$\omega_2 = 4\left(\frac{\pi}{L}\right)^2\sqrt{\frac{gEI}{w}}$

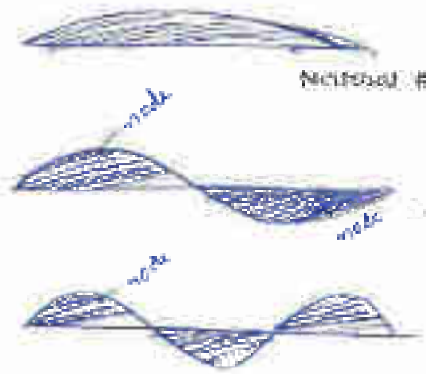
$\omega_1 = \omega_n$

$\omega_2 = 4\,\omega_n$
$= 2^2\,\omega_n$

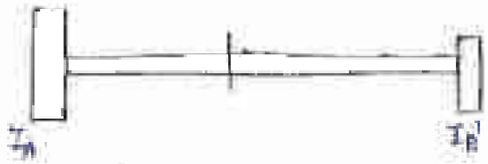
$\boxed{\omega_2 = 2^2\,\omega_n}$ — 2-mode

$\omega_3 = 9\,\omega_n$

$\boxed{\omega_3 = 3^2\,\omega_n}$ — 3-mode



CRO-46



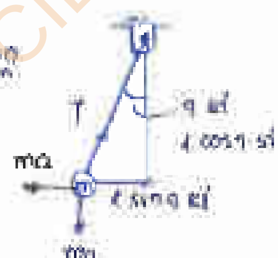
$\frac{\omega_B}{\omega_A} = n$ (given)

$\rightarrow \frac{1}{2} I_B' (\omega_B')^2 = \frac{1}{2} I_B (\omega_B)^2$

$I_B' (\omega_A^2) = I_B (\omega_B^2)$

$\boxed{I_B' = I_B\, n^2}$

CRO-47



$ma \times l \cos 9\,\text{rel} = mg\,l \sin 9\,\text{rel}$

$a = g \tan 9\,\text{rel}$

$\boxed{a = 1.66\ m/s^2}$

CRO-50 two nodes ⟶ 3-mode

$\omega_3 = (3)^2 \omega_1$ (nodes) $\Rightarrow 1800 = 3^2 \omega_1$

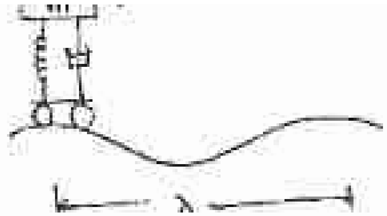
$\boxed{\omega_1 = 200\ rpm}$

CRO 48

$v = \lambda f$

$f = v/\lambda$

$\rightarrow \omega = 2\pi f \Rightarrow \omega = \frac{2\pi v}{\lambda}$



@ Resonance

$\omega = \omega_n$

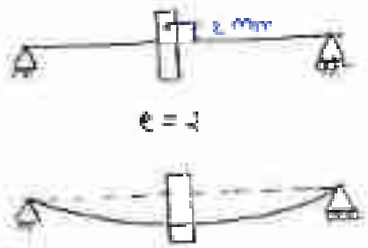
$\frac{2\pi v}{\lambda} = \sqrt{\frac{k}{m}}$

CRO 49

Shaft frequency → natural frequency

$\omega_n = 10$ Rad/s $= \omega_1$

$e = 2$ mm



$\rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60}$

$= 31.4$ Rad/s

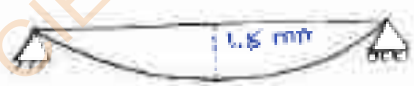
$\Rightarrow y = \frac{e}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} = \frac{2}{\left(\frac{31.4}{10}\right)^2 - 1} = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} = \frac{2}{\left(\frac{10}{31.4}\right)^2 - 1}$

$y = -2.25$ mm

CRO - 51

Short bearing = simply supported

long bearing = fixed end



$\rightarrow \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{9.81}{1.8 \times 10^{-3}}} \Rightarrow \omega_n = 705$ Rpm

52

Gate-18

$N_{oper} = 1440$ rpm

solid shaft $= D$

→ Critical speed is enhanced

$D_o = D$,

$D_i = 0.75 D$

→ For end cond^n use SSB

$\omega_n = \sqrt{\frac{K}{m}}$

$K = \frac{48EI}{l^3}$

$\delta = \frac{Wl^3}{48EI} \Rightarrow \delta = \frac{K \delta l^3}{48EI}$

$\boxed{\omega_n \propto \sqrt{I}}$

$\Rightarrow \frac{\omega_{n1}}{\omega_{n2}} = \sqrt{\frac{I_1}{I_2}} \Rightarrow \frac{1440}{N_2} = \sqrt{\frac{\pi/64\, D^4}{\pi/64\,[D^4 - (0.75D)^4]}}$

$\frac{1440}{N_2} = \sqrt{\frac{1}{1-(0.75)^4}}$

$\boxed{N_2 = 1157.5 \text{ RPM}}$

→ Hollow shaft use is comfortable

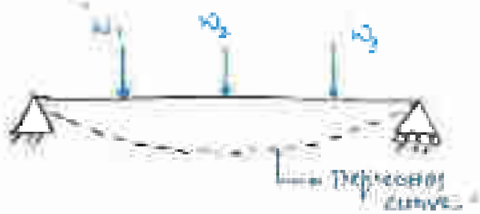
→ since by using Hollow shaft $(N_{op} - N_{cri})$ is increasing, we can comfortably accelerate the shaft. Therefore using Hollow shaft then solid will be a good alternative.

→ since, critical speed decrease, so you can't to win this method

When shaft is having number of masses

(2) Dunkerley's method :

$$\frac{1}{(\omega_n)^2} = \frac{1}{\omega_{n_1}^2} + \frac{1}{\omega_{n_2}^2} + \frac{1}{\omega_{n_3}^2} + \dots$$



$\rightarrow$ $\omega_{n_1} = \sqrt{\frac{g}{\delta_1}}$ → deflection @ the point load W_1, when W_1 is acting alone

⇒ Simply Supported Beam :

1)

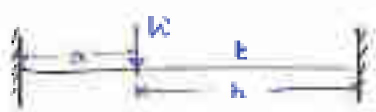
$$\delta = \frac{W a^2 b^2}{3 E I l}$$

@ $a = b = \frac{l}{2}$ ⇒ $\delta = \frac{W l^3}{48 EI}$

2.

$$\delta = \frac{5}{384} \frac{w l^4}{EI}$$ @ mid span

3. fixed beam



$$\delta = \frac{W a^3 b^3}{3 E I L^3}$$ @ point load

$a = b = L/2$,

$$\delta = \frac{W l^3}{192 EI}$$ @ mid point

$$\delta = \frac{wl^4}{384\,EI}$$ @ mid span

⇒ Cantilever.

$$\delta_{\text{free end}} = \frac{Wl^3}{3EI}$$

$$\delta_{\text{free end}} = \frac{wl^4}{8EI}$$

* Gear : The larger wheel is known as gear.
* Pinion : The smaller wheel is known as pinion.
- Due to smaller in size, smaller of pinion is less than ~~that~~ ~~of~~ gear that is why it is driver.
  pinion is driver (in general)

→ Velocity Ratio

$$V.R = \frac{\omega_{dr}}{\omega_{dn}} \quad (>1)$$

$$= \frac{\omega_{dn}}{\omega_{dr}} \quad (<1)$$

→ Gear Ratio

$$\text{Gear ratio} = \frac{T}{t} \quad (>1)$$

$$= \frac{t}{T} \quad (<1)$$

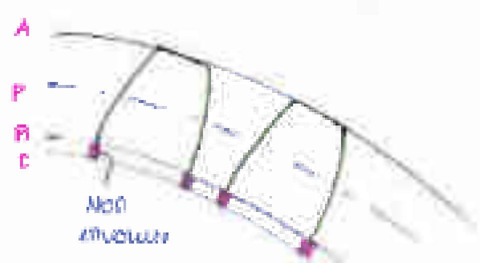
* Gear terminology

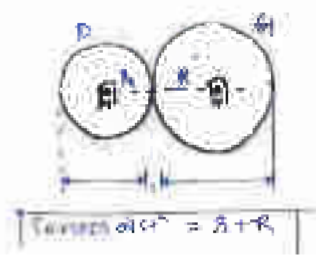
(1) Pitch circle : It is an imaginary circle (whose is the radius = be changed)
It is the most important circle in gears as it specifies the size of gear and all the dimensions of gear are measured along the pitch circle only.

(2) Base circle : It is the smallest circle on gear from where the envolute profile is begins.
- It is a real circle and it's radius can not be change.
- Common normal in gear which is also known as line of action is tangential to both the base circles (gear and pinion).

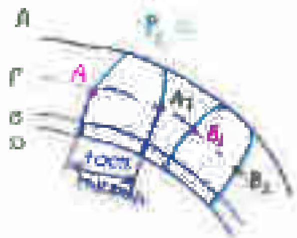
(3) Dedendum circle (m) / Root circle : It is smallest circle from where the [illegible] begins. Small circle

(4) Addendum circle : A circle which passes through the top of gear tooth.





→ **Circular pitch** : The distance between two similar points on adjacent teeth measured along the pitch circle circumference. (A_1B_1) (A_2B_2)



$$P_c = \frac{\text{pitch circle circumference}}{\text{no. of teeth}}$$

for gear $P_c = \frac{\pi D}{T}$

$P_c = \frac{\pi d}{t}$

→ **Diametrical pitch** : No. of teeth per inch diameter (It is FPS unit)

$$P_d = \frac{\text{no. of teeth}}{\text{p.c.d}}$$

for gear $P_d = \frac{T}{D}$

$P_c = \frac{\pi}{P_d}$

→ **Module** : It is SI unit of gear defined as

$$m = \frac{\text{p.c.d}}{\text{no. of teeth}}$$

for gear $m_{gear} = \frac{D}{T}$ (mm)

$m_p = \frac{d}{t}$

NOTE : Two gears which are in mesh have same unit.

If gear and pinion are in mesh

$m_{gear} = m_{pinion}$

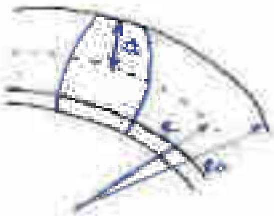
$\frac{D}{T} = \frac{d}{t}$ ⇒ $\frac{D}{d} = \frac{T}{t}$

(6) **Dimensions of gears**

i) **Tooth thickness** : The thickness of tooth measured along pitch circle circumference (A_1A_2) or (B_1B_2)

ii) **Tooth space** or **space width** : The distance between two successive teeth measured along pitch circle circumference (A_2B_1)

iii) Addendum / Addenda: [illegible] and pitch circle is known as addenda



for gear $a = R_a - R$

for pinion $a = r_a - r$

$a = f \cdot m$ → fraction

$a = 1$ module → for full depth

$a = 0.8$ module → for stub tooth

- The wheel with larger addendum always decide the beginning of engagement

iv) Dedendum / Dedenda: The radial dist^n between pitch circle & dedendum circle

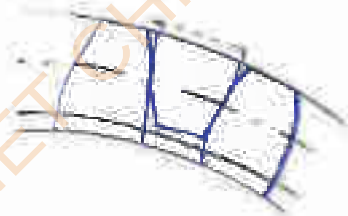
$b = R - R_d$ (for gear)

$b = r - r_d$ (for pinion)

(v) Full depth: The summation of $(a + b)$ i.e. addendum and dedendum called full depth

(vi) Working depth: Summation of addendum of gear & pinion is known as working depth

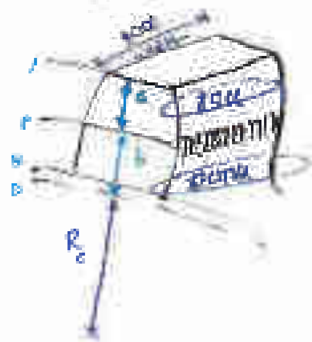
- In order to avoid interference working depth should be less than full depth



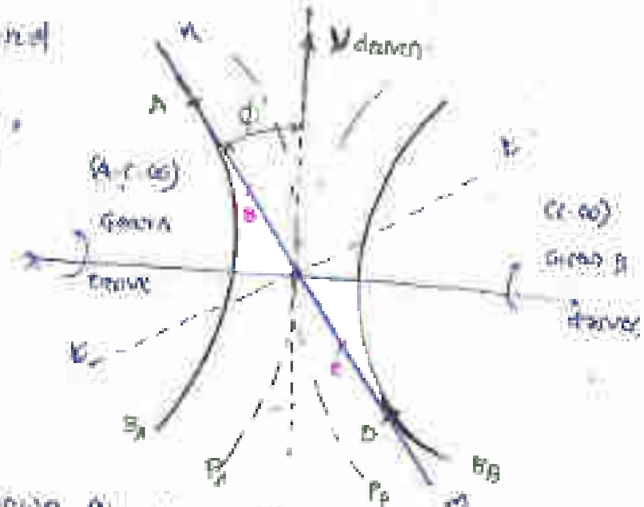
(vii) Face: The portion of tooth above the pitch surface known as face

(viii) Flank: The portion of tooth below pitch surface is flank

(ix) Pressure angle: It is measure of [illegible] of gear and pinion mechanism



- pressure angle is denoted by ϕ. Gear set-
- This the angle between direction velocity vector of driven gear to the common normal.
- The angle between common tangent to both the pitch circle and common normal is known as pressure angle.



— common normal is known as line of action. It will be tangent to both the base circles.
— Force transmission, path of contact etc. always take place along the common normal.

⇒ Clearance

Types of clearance

1) circumferential clearance ⇒ Backlash
2) Radial clearance ⇒ clearance

- Backlash appears in meshing gears
- It is the amount by which tooth space is greater than teeth thickness of mating gears.
- Backlash always provided for following reason
  1) During meshing thermal expansion of gear may take place & backlash take care of this
  2) Backlash take care of machining allowance
  3) Backlash can be increased by increasing the center dist^n, & does not affect velocity ratio

- The distance between addendum of gear and dedendum of pinion or vice versa is known as clearance.
- clearance is amount by which gear depth exceeds the working depth.

⇒ clearance is always provided always in order to provide non conjugate action, that is meshing of involute profile with non involute profile, most commonly known as

⇒ Length of path of contact.

→ case (i)

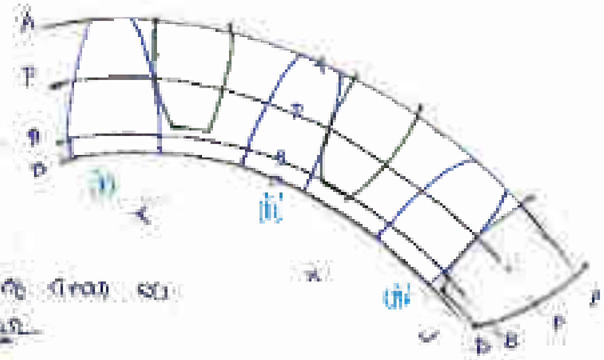
Involute profile of Gear-A is meshing with Gear-B so interference will not occur.

→ case (ii).

Gear-B is just meshing or touching the base circle of Gear so interference will not occur.

→ case (iii)

Involute profile of Gear B crosses the base circle boundary of Gear A so then interference occur.



– BF → path of approach

– CP → path of recess

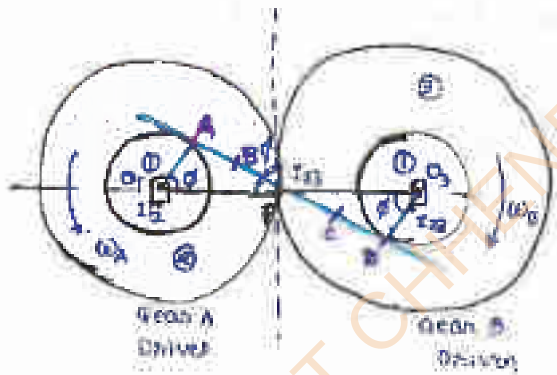
Law of gearing :

Gear A → Link 2

B → 3

Angular vel. theorem

$$\frac{\omega_2}{\omega_3} = \frac{I_{13} I_{23}}{I_{12} I_{23}} = \frac{O_3 P}{O_2 P}$$



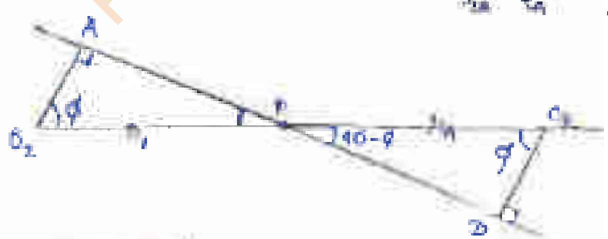
$$\boxed{\frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} = \frac{t_B}{t_A}} \quad \text{— (1)}$$

$m_A = m_B$ & $\frac{d_A}{t_A} = \frac{d_B}{t_B}$

$\frac{r_B}{r_A} = \frac{t_B}{t_A}$

In $\triangle O_2AP$ & $\triangle O_3BP$

$$\frac{O_2 P}{O_3 P} = \frac{AP}{BP} = \frac{O_2 A}{O_3 B}$$



Since $\frac{O_2 A}{O_3 B}$ = const.

Hence $$\boxed{\frac{\omega_B}{\omega_A} = \frac{O_2 P}{O_3 P} = \frac{AP}{BP} = \frac{O_2 A}{O_3 B} = \text{const}}$$

Statement of Law of gearing

- Velocity ratio in gears always remains constant
- The pitch point P which is the point of contact of pitch circles is a fixed point on a line O_1O_2 will be on centers I_{23}
- The pitch point P divides the centre distance O_1O_2 in a constant ratio.
- The common normal (Line of Action) passes through pitch point and the pitch point divides it in a constant ratio.
- Presence of teeth is not a necessary condition to call an element of gear.
- If two cylinders which does not have teeth on them but satisfy the law of gearing, we will call them as gears.

@ pitch point Gear A & Gear B are in pure rolling
@ pitch point : velocities are same

$$r_A\omega_A = r_B\omega_B \Rightarrow \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A}$$

@ pitch point velocity of sliding is zero

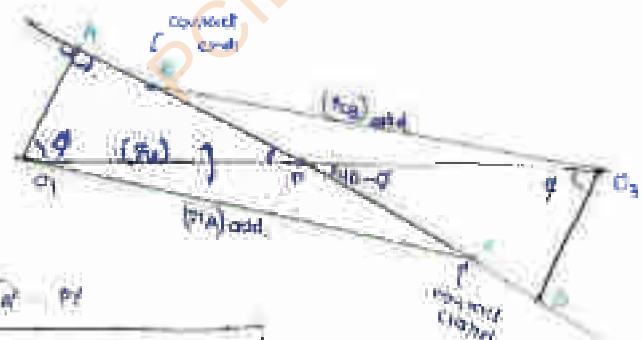
- $AP = r_A \sin\phi$ & $PD = r_B \sin\phi$

$(r_b)_A = O_1A = r_A \cos\phi$; $O_2D = r_B \cos\phi$

pitch circle radius

Base circle radius = [pitch circle radius] × cos φ

→ Path of contact = path of approach + path of recess = CP + PB



- path of approach = CP = CA − PA

$= \sqrt{O_1C^2 - O_1A^2} - PA$

$CP = \sqrt{(r_A)_{add}^2 - (r_A\cos\phi)^2} - r_A \sin\phi$

- path of recess = PB = BD − PD

$PB = \sqrt{(r_B)_{add}^2 - (r_B\cos\phi)^2} - r_B \sin\phi$

Path of contact $= \left[\sqrt{(r_{A,a})^2 - (r_A \cos\phi)^2} - r_A \sin\phi\right]$
$+ \left[\sqrt{(r_{B,a})^2 - (r_B \cos\phi)^2} - r_B \sin\phi\right]$

⇒ max^n path of contact = AD

⇒ max^n path of approach = BP = $r_B \sin\phi$

⇒ max^n path of recess = PA = $r_A \sin\phi$

max^n path of contact $= (r_A + r_B)\sin\phi$

Arc of contact $= \dfrac{\text{Path of contact}}{\cos\phi}$

Contact ratio $= \dfrac{\text{Arc of contact}}{\text{Circular pitch (P_c)}}$

→ Contact ratio predicts the No. of pairs of tooth which are in mesh for a smooth operation contact ratio
$1 < CR < 2$

→ If $CR = 1$ ⇒ it means one pair of tooth in mesh at pitch point.

→ If $CR = 1.2$ ⇒ It means one pair of tooth is in complete meshing at pitch point and another pair of tooth is mesh for 20% of total arc of contact.

⇒ Velocity of sliding
@ beginning of engagement $= (l_{PC})(\omega_p + \omega_g)$
$= PC\,(\omega_p + \omega_g)$
$=$ path of approach $\times (\omega_p + \omega_g)$

⇒ Velocity of sliding
@ end of engagement $= (l_{PA})(\omega_p + \omega_g)$
$=$ path of recess $\times (\omega_p + \omega_g)$

→ Velocity of sliding
@ pitch point

@ Angle of action (δ):

$\delta_{gear} = \dfrac{\text{arc of contact}}{R}$

$\delta_{pinion} = \dfrac{\text{arc of contact}}{r}$

Defⁿ. Angle of action: The angle turned through by the gear or pinion along pitch circle either gear or pinion T the are always subtends angle at centre known as angle of action.

Defⁿ Interference: Whenever involute profile mates with non involute profile it result in non conjugate action known as interference.

- Interference leads to several problem like jamming, etc. therefore it must be avoided.

⟹ Methods to avoid interference:

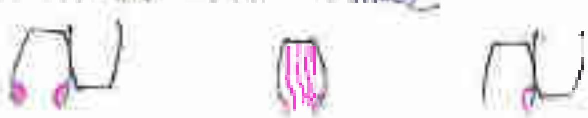
1) By changing the center distance
- If we increase the center distⁿ, pressure angle increases automatically.
- On increase the center distance, pressure angle increases. Hence, interference can be avoided.

2) Stubbing the tooth
Stubbing means removal of actual portion from the top of gear tooth.
- Stubbing always decreases the strength of gear tooth since movement arm reduces.

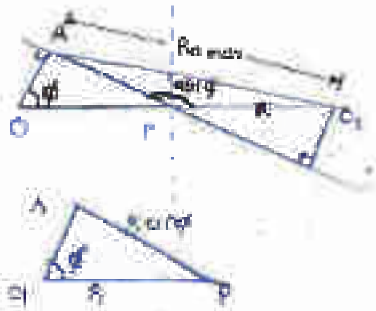
3) By using cycloidal teeth.
- In cycloidal tooth conjugate action is always maintain due to which interference does not takes place.

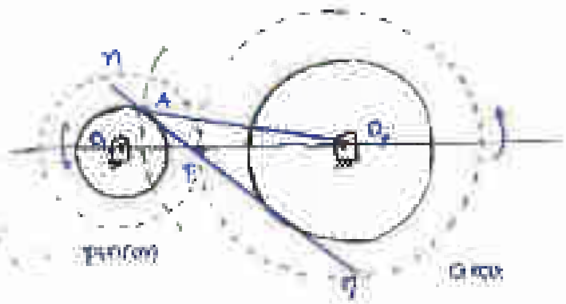
4) By properly choosing the no. of teeth on pinion

5) Undercutting
- During manufacturing the cutter (tool) removes some material from the flank known as undercutting.
- During machining if interference takes place, the involute portion of gear tooth tries to remove the non involute portion if it is able to remove it be known as under cutting.
- Some times we purposely remove some material from the flank so that free space becomes available for the gear tooth to mesh properly w/o under going in interference is known undercutting.
- Undercutting always result in stress concentration, which will make the tooth weaker.



→ [illegible]





In O_1PA,

$$(O_1A)^2 = (O_1P)^2 + (PA)^2 - 2(O_1P)(PA)\cos(90+\phi)$$

$$\Rightarrow R_{r,max}^2 = R^2 + (r\sin\phi)^2 + 2Rr\sin\phi\sin\phi$$

$$R_{r,max} = \sqrt{R^2 + r^2\sin^2\phi + 2Rr\sin\phi\sin\phi}$$

$$= \sqrt{R^2\left[1 + \frac{r^2}{R^2}\sin^2\phi + \frac{2r}{R}\sin^2\phi\right]}$$

$$\frac{r}{R} < 1 \Rightarrow \boxed{\lambda = \frac{r}{R} = \frac{\omega_g}{\omega_p}} \quad (\lambda < 1)$$

$$\Rightarrow R_{a,max} = R\sqrt{1 + \lambda(\lambda+2)\sin^2\phi}$$

$$\Rightarrow R_{a,max} - R = R\sqrt{1 + \lambda(\lambda+2)\sin^2\phi} - R$$

$$\Rightarrow a_{gear,max} = R\left[\sqrt{1 + \lambda(\lambda+2)\sin^2\phi} - 1\right]$$

(★) $$f\,m_{(min)} = R\left[\sqrt{1 + \lambda(\lambda+2)\sin^2\phi} - 1\right]$$

$$\Rightarrow f\,\frac{2R}{t_{min}} = R\left[\sqrt{1 + \lambda(\lambda+2)\sin^2\phi} - 1\right]$$

$$\Rightarrow t_{min} = \frac{2f}{\left[\sqrt{1 + \lambda(\lambda+2)\sin^2\phi} - 1\right]}$$

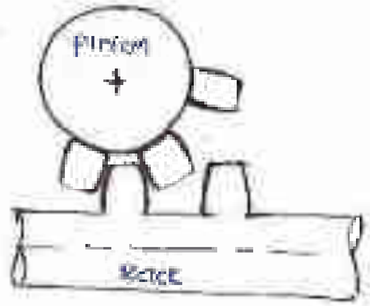
$$\boxed{t_{min} = \frac{2f}{\left[\sqrt{1 + \lambda(\lambda+2)\sin^2\phi} - 1\right]}} \qquad \lambda = \frac{r}{R}$$

→ Actual teeth ($t_{act} > t_{min}$) must be greater than t_{min} to avoid interference.

➔ Minimum no. of teeth on pinion to avoid interference in **Rack & pinion** arrangement

$R \to \infty$

$\lambda = \frac{z}{R} \to$ $\boxed{\lambda = 0}$



$$t_{min} = \lim_{\lambda \to 0} \frac{2f\lambda}{\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1} \quad \left(\frac{0}{0}\right)$$

L- Hospital Rule

$$t_{min} = \lim_{\lambda \to 0} \frac{\frac{d}{d\lambda}(2f\lambda)}{\frac{d}{d\lambda}\left[\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1\right]}$$

$$= \frac{2f}{\frac{1}{2\sqrt{1+\lambda(\lambda+2)\sin^2\phi}}\{\lambda + (\lambda+2)\}\sin^2\phi} = \frac{2f}{\frac{2\sin^2\phi}{2\sqrt{1+0}}}$$

$$\boxed{t_{min} = \frac{2f}{\sin^2\phi}}$$

NOTE :- If $a = 1$ module i.e. $f = 1$

V.R ⇒ $\boxed{\lambda = 1}$ ⇒ $\boxed{z = R}$

gear & pinion are same size

$$t_{min} = \frac{2f\lambda}{\sqrt{1+\lambda(\lambda+2)\sin^2\phi} - 1}$$

$$\boxed{t_{min} = \frac{2}{\sqrt{1+3\sin^2\phi} - 1}}$$

Remember

ϕ	t_{min}
14.5°	23
20°	13
22.5°	11

} For gear & pinion $\boxed{f = 1}$

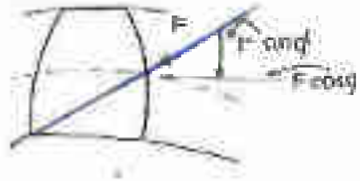
ϕ	t_{min}
14.5°	32
20°	18
22.5°	14

} For rack & pinion $\boxed{f = 1}$

→ F cosφ component is responsible for power in gear.

P ∝ F cosφ

If φ ↑ , cos φ ↓
power ↓
interference ↑↓



φ	interference	power
14.5°	more	max"
20°	moderate	moderate
22.5°	min	min

⇒ Limitation of method to avoid interference :-

1) Center distance between gears is the center distance b/w shaft on which they are mounted & distance b/w shaft can not changed.

2) By stubbing the teeth length of path of contact decreased which result in decrement of contact ratio because of which operation becomes non smooth.

Involute

1) Locus of a point on straight line that rolls without slipping on a circle.

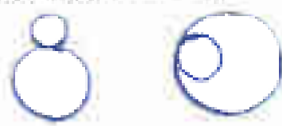
2) Involute is also obtained when tightened string is unwrapped from a pulley.

3) Due to non conjugate action involute cutting are interference.

Cycloid

1) It is locus of a point on a circle that rolls w/o slipping on a straight line.

2) cycloid = Epicycloid + Hypocycloid



3) GATE

In cycloidal epicycloidal meshes with hypocycloid & hypocycloid meshes with epicycloid, therefore non conjugate action ...

- Pressure angle in involute is always constant

- Less costly & easy to manufacture

does not undergo ...

- In cycloidal pressure angle is max at beginning of engagement & zero at pitch point again max at the end of engagement

- Difficult to manufacture & more costly

Ex- In watches cycloidal tooth used

Note → In involute gear even if centre distⁿ is changed, velocity ratio remain constant

Note → Worm gear is used to obtain a large velocity ratio

(20-2)

$m = 4$

$T = 32$

→ $m = \frac{D}{T}$

$D = 32 \times 4 = 128$ mm

∴ Tooth thickness = tooth space

32 tooth thickness + 32 tooth space = pitch circle circumference

64 tooth thickness = πD

Tooth thickness $= \frac{\pi D}{64}$

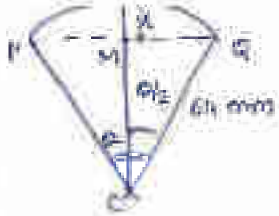
Tooth thickness $= \frac{128 \pi}{64}$

| Tooth thickness = 6.28 mm | = arc (PQ)

angle $= \frac{\text{arc}}{\text{Radius}}$

$\theta = \frac{6.28}{64}$

$\theta = 5.62°$



L → is working framing

$PQ = PM + MQ$

$= 2MQ = 2 \times 64 \sin \theta/_2$

$\boxed{PQ = 5.27 \text{ mm}}$

→ $x = CN - CM = 64 - 64 \cos \theta/_2$

$= 0.077$

$b = \text{addendum} + x$

$= 1m + x = 4 + 0.077$

$\boxed{b = 4.1 \text{ mm}}$

Q^n - 2 teeth on pinion $t = 16$

$m = 5$ mm

dedendum $= 1m$

What should be pressure so that dedendum remains of purely an involute profile. (?)

→ For dedendum consist purely involute profile
dedendum circle are base circle

Radius of dedendum circle = Radius of base circle

$r - \text{dedendum} = r \cos\phi$

$40 - 5 = 40 \cos\phi$

$\boxed{\phi = 29°}$

$m = \frac{D}{t} = \frac{2r}{t} = 5$

$\boxed{r = 40}$

Ex Two meshing spur gears with 20° pressure angle have module 4 mm, centre distance 220 mm. No. of teeth on pinion is 40 so what value should the centre dist^n be increased so that pressure angle becomes 22° (?).

→ Central dist^n $r + R = 220$

$m = \frac{d}{t} \Rightarrow 4 = \frac{d}{40}$

$d = 160$

$\boxed{r = 80}$

$\boxed{R = 140}$

$\phi = 20$,
$m = 4$
$C.d = 220$
$t = 40$
$\phi' = 22°$

→ $r_b = r \cos\phi = (80) \cos 20$

$\boxed{r_b = 75.175 \text{ mm}}$

→ $R_b = R \cos 20 = (140) \cos 20$

$\boxed{R_b = 131.55 \text{ mm}}$

In, $r_b = r' \cos\phi' \Rightarrow 75.175 = r' \cos 22°$

$\Rightarrow \boxed{r' = 81.078 \text{ mm}}$

$R_b = R' \cos\phi' \Rightarrow 131.556 = R' \cos 22°$

$\Rightarrow \boxed{R' = 141.88 \text{ mm}}$

→ New center dist^n $= r' + R'$

$= \underline{222.93} \text{ mm}$

$(VR)_i = \frac{R}{r} = \underline{1.75}$

$(VR)_f = \frac{R'}{r'} = \underline{1.75}$

Note → In involute by changing the center distance velocity ratio does not change in involutes.

Ex: Two gears are in mesh have module of 10 mm and pressure angle of 25°, pinion has 20 teeth and gear has 52 teeth. Addendum of both gear is equal to 1 m. determine

(i) No. of pairs of teeth in contact

(ii) angle of action of the pinion and gear

(iii) Ratio of sliding velocity to rolling velocity at beginning of engagement at the pitch point and at the end of engagement

→ $m = 10$ mm

$\phi = 25°$

$T = 52$

$t = 20$

$a = 1m = 10$ mm

$m = \frac{d}{t} \Rightarrow d = 10 \times 20$

$= 200$ mm

$\boxed{r = 100 \text{ mm}}$

$m = \frac{D}{T} \Rightarrow D = 52 \times 10$

$\boxed{R = 260 \text{ mm}}$

$R_a = R + a = 260 + 10$

$\boxed{R_a = 270 \text{ mm}}$

$r_a = r + a = 100 + 10$

$\boxed{r_a = 110 \text{ mm}}$

→ Gear & pinion have same module

Path of approach $= \sqrt{R_a^2 - (R\cos\phi)^2}$

path of approach = $\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{(110)^2 - (100\cos 20)^2} - 100\sin 20$

$= 20.07$ mm

→ path of recess $= \sqrt{R_r^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{270^2 - (260\cos 20)^2} - (260\sin 20)$

$= 21.928$ mm

(1) path of contact = P.O.A + P.O.R

$= 20.07 + 21.928$

$= 42$ mm

(2) contact ratio = $\dfrac{\text{Arc of contact}}{\text{Circular pitch}}$

{ Arc of contact $= \dfrac{\text{path of contact}}{\cos\phi}$

$= \dfrac{42.004}{\cos 20\,(\pi \times 10)}$

$= 1.475$ (one pair of teeth in complete mesh at pitch pt. for 47.5% of total time contact with ...)

(3) velocity @ beginning engagement = path of approach $(\omega_p + \omega_g)$

$= P.O.A\,(\omega_p + \omega_g)$

$\dfrac{V_{sliding}}{V_{rolling}}$ @ beginning engagement $= \dfrac{P.O.A\,(\omega_p + \omega_g)}{r\,\omega_p}$

$= \dfrac{POA}{r}\left(1 + \dfrac{\omega_g}{\omega_p}\right)$ { $\dfrac{\omega_p}{\omega_g} = \dfrac{R}{r}$

$= \dfrac{20.7}{100}\left(1 + \dfrac{100}{260}\right)$

$= 0.2866$

angle of action

(4) $\delta_{gear} = \dfrac{\text{Arc of contact}}{R} = \dfrac{46.347}{260}$

$= 0.1782$ rad

$\delta_{pinion} = \dfrac{\text{arc of contact}}{r} = \dfrac{46.347}{100}$

$= 0.4634$ rad

$\dfrac{V_{sliding}}{V_{rolling}}\Big|_{@\ pitch\ point} = 0$

$\dfrac{V_{sliding}}{V_{rolling}}\Big|_{@\ end} = \dfrac{\text{Path of recess } (\omega_p + \omega_g)}{R\omega_p}$

$= \dfrac{41.9732}{100}\left[1 + \dfrac{\omega_g}{\omega_p}\right] = \dfrac{41.91}{100}\left[1 + \dfrac{R}{R}\right]$

$= 0.103$

Two 20° involute spur gears having module of 6 mm, the larger wheel has 36 teeth, the pinion has 16 teeth if addendum have 1 m. Will the interference occur? What will happen if the no of teeth reduced to 14 teeth

$\phi = 20°$
$m = 6$
$T = 36$
$t = 16$
$m = 1\,m$

$\rightarrow m = \dfrac{D}{t} \Rightarrow 6 = \dfrac{d}{16}$

R = 48

$6 = \dfrac{D}{36} \rightarrow$ **R = 108**

→ No. of min of teeth on pinion

$\lambda = \dfrac{r}{R} = \dfrac{t}{T} = 0.44$

$f = 1$ to avoid interf.

$t_{min} = \dfrac{2f\lambda}{\sqrt{1 + \lambda(\lambda+2)\sin^2\phi} - 1} = \dfrac{2 \times 1 \times 0.44}{\sqrt{1 + 0.44(0.44+2)} - 1}$

$t_{min} = 15$

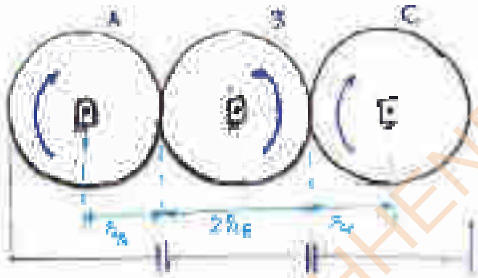
→ min teeth is 15 and $t_{act} = 16$ Hence interference will not occur. ⇒ Cond ($t_{act} > t_{min}$) ⇒ to avoid interference if the act. no of teeth reduced to be 14 which is less than t_{min} req" ⇒ interference will occur

Gear train

- DOF = 1
  - each gear on different shaft → simple gear train
  - two gears are on same shaft → compound gear train → Inverted gear train
- DOF = 2 → sun & planet gear train (or) Epicyclic gear train

⇒ **Simple Gear train**

- In simple gear train all the gears are meshing along on the same plan they are used to connect the shaft away from each other.



- If the center distance between the shaft is large belt & pulley arrangement is required over simple gear train, since the efficiency of gears is larger than belt & pulley, although there is slipping also.

$$\text{center distance} = r_A + 2r_B + r_C$$

- If even no of gear train are used then i/p & o/p are rotate in opposite direction.
  If odd no of gears are used then i/p & o/p rotate in same direction.

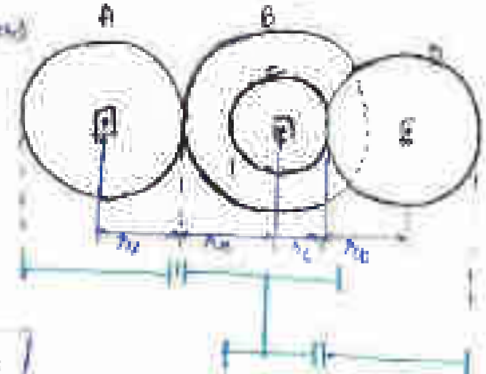
Let A/B/C/D gears present

$$\frac{\omega_A}{\omega_D} = \frac{\omega_A}{\omega_B}\cdot\frac{\omega_B}{\omega_C}\cdot\frac{\omega_C}{\omega_D} = \frac{T_B}{T_A}\cdot\frac{T_C}{T_B}\cdot\frac{T_D}{T_C}$$

$$T.V = \boxed{\frac{\omega_A}{\omega_D} = \frac{T_D}{T_A}}$$

-- The gears which does not decide the magnitude of train value are known as idlers gear. They are affect only direction.

- B & C are compound gears
- compound gear train is used to connect the input which are parallel & input & output gears will arrange exist on parallel planes



Centre distance $= r_A + r_B + r_C + r_D$

T.V :

$$T.V = \frac{\omega_A}{\omega_B} = \frac{\omega_A}{\omega_B} \frac{\omega_B}{\omega_C} \frac{\omega_C}{\omega_D}$$

{ B & C are same shaft

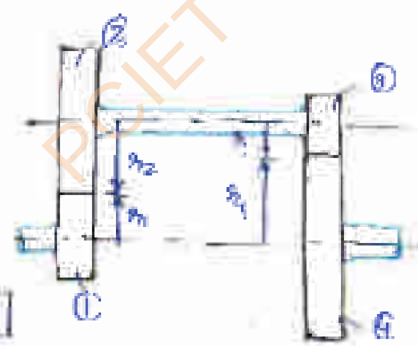
$$= \frac{\omega_A}{\omega_B} \cdot \frac{\omega_C}{\omega_D}$$

$$T.V = \boxed{\frac{\omega_A}{\omega_D} = \frac{T_B}{T_A} \cdot \frac{T_D}{T_C}}$$

T.V = product of no of teeth on driven gears / product of no of teeth on driver gears

→ **Inverted train** (or) **Reverted train**

- If the input & output shaft are co-linear & they are connected by reverted gear train

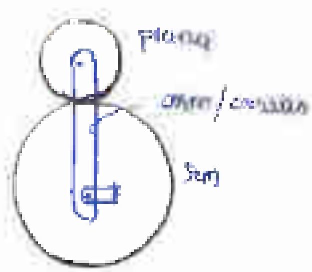


$r_1 + r_2 = r_3 + r_4$

$\Rightarrow T_1 + T_2 = T_3 + T_4$

→ **Epicyclic Gear Train** (or) sun & planet gear train

DOF = 2



(b) $D_Q = 2D_R$

$m_R = 2$

$m_R = \frac{D_R}{T_R} \Rightarrow 2 = \frac{D_R}{15}$

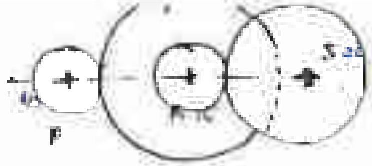
$\boxed{D_R = 30}$

$m_S = \frac{D_S}{T_S} \Rightarrow \boxed{D_S = 40}$

$\rightarrow D_Q = 2(D_R) = 60 \Rightarrow \boxed{D_Q = 60}$

$\rightarrow m_P = m_Q \Rightarrow \frac{D_P}{T_P} = \frac{D_Q}{T_Q} \Rightarrow \frac{D_P}{20} = \frac{60}{40}$

$\boxed{D_P = 30}$



$C.D = R_P + R_Q + R_R + R_S$

$= 15 + 30 + 15 + 20$

$= 80$ mm

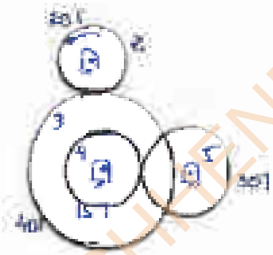
(11)

$\frac{N_2}{N_5} = \frac{N_2}{N_3} \times \frac{N_3}{N_4} \times \frac{N_4}{N_5}$ $\left\{ N_3 = N_4 \right.$ same shaft

$= \frac{N_2}{N_3} \times \frac{N_4}{N_5}$

$= \frac{T_3}{T_2} \times \frac{T_5}{T_4} = \frac{40}{20} \times \frac{30}{15}$

$\frac{1200}{N_5} = 4 \times \Rightarrow \boxed{N_5 = 300 \text{ rpm}}$ (a)



(10) The T.V eq^n which even tooth not come in eq^n is idler.

$\frac{N_2}{N_6} = \frac{N_2}{N_3} \times \frac{N_3}{N_4} \times \frac{N_4}{N_5} \times \frac{N_5}{N_6}$

$= \frac{T_3}{T_2} \times \frac{T_4}{T_4} \times \frac{T_6}{T_5}$

$= \frac{T_3}{T_2} \times \frac{T_6}{T_5}$ $\left\{ T_4 \right.$ is not come in T.V eq idlers

(13)* [C]

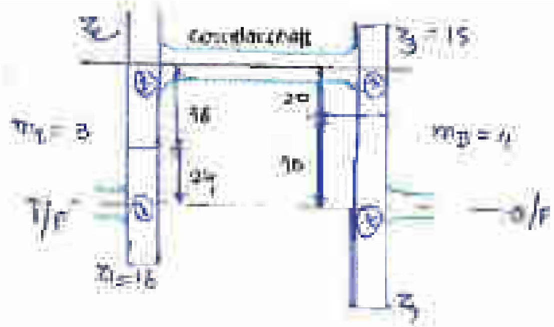
$G.R = \frac{\text{No. of teeth gear}}{\text{No. of pinion}}$

$i = \frac{Z_2}{Z_1}$

Given, $G.R = 4$

$m_I = 3$, $m_{II} = 4$

overall $G.R = 12$



→ $\frac{Z_1}{Z_2} = \frac{1}{4}$ ⇒ $16 \times 4 = Z_2$ ⇒ $\boxed{Z_2 = 64}$

→ $m_I = \frac{D_1}{Z_1}$ ⇒ $D_1 = 3 \times 16 = 48$ ⇒ $\boxed{R_1 = 24}$

→ $m_I = \frac{D_2}{Z_2}$ ⇒ $D_2 = 3 \times 64 = 192$ ⇒ $\boxed{R_2 = 96}$

→ $m_{II} = \frac{D_3}{Z_3}$ ⇒ $D_3 = 4 \times 15$ ⇒ $\boxed{R_3 = 30}$

$C.C = R_1 + R_2 = 24 + 96 = 120$ ⇒ $\boxed{C.C = 120}$

⇒ from $G.R = 12$ → (total)

$\frac{\omega_{I/P}}{\omega_{O/P}} = 12$ ⇒ $\frac{\omega_1}{\omega_4} = 12$

$\frac{\omega_1}{\omega_2} \times \frac{\omega_2}{\omega_3} \times \frac{\omega_3}{\omega_4} = 12$

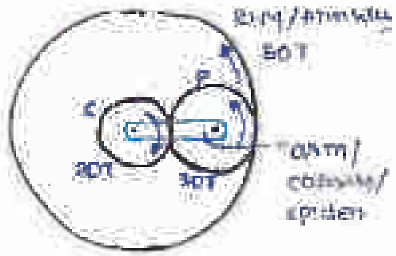
$\frac{Z_2}{Z_1} \times \frac{Z_4}{Z_3} = 12$ ⇒ $4 \times \frac{Z_4}{15} = 12$

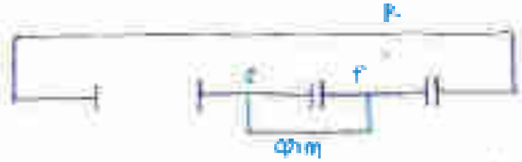
$\boxed{Z_4 = 45}$

[14]

Sun - planet gear

Two 14





$\left\{ \frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} \right.$

Condition	Arm	20 Gear S	30 Gear P	80 Gear R
Arm is fixed, Gear S rotating with (+x) rpm (CCW)	0	+x	$-x \cdot \left(\frac{20}{30}\right) = -\frac{2x}{3}$	$-\frac{2x}{3} \cdot \left(\frac{30}{80}\right) = -\frac{x}{4}$
Arm rotating with +y rpm	y	y + x	$y - \frac{2x}{3}$	$y - \frac{x}{4}$

→ Ring gear fixed

$y - x/4 = 0 \Rightarrow \boxed{x = 4y}$

→ Arm Sun speed $N_{sun} = 100$

$y + x = 100 \Rightarrow 5y = 100$

$\boxed{y = 20}$ (CCW)

Shortcut

$\frac{\omega_S}{\omega_R} = -\frac{\omega_S}{\omega_P} \cdot \frac{\omega_P}{\omega_R}$

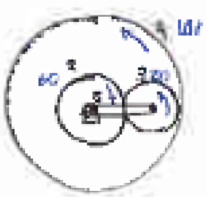
$\frac{\omega_S - \omega_{arm}}{\omega_R - \omega_{arm}} = -\frac{T_P}{T_S} \cdot \frac{T_R}{T_P} =$

$\frac{100 - \omega_{arm}}{0 - \omega_{arm}} = -4$

$\boxed{\omega_{arm} = 20}$

(If fell rotating opposite dirn take (-ve))

22



Shortcut

$\frac{\omega_2}{\omega_4} = -\frac{\omega_2}{\omega_3} \times \frac{\omega_3}{\omega_4}$

$\frac{\omega_2 - \omega_{arm}}{\omega_4 - \omega_{arm}} = -\frac{T_3}{T_2} \times \frac{T_4}{T_3} = -\frac{20}{60} \times \frac{100}{20}$

[illegible] $\frac{0 - \omega_{arm}}{100 - \omega_{arm}} = -\frac{10}{6}$

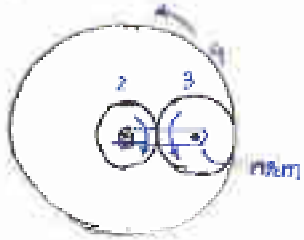
Condition	Arm
Arm is on 2nd Gear	

$6\,\omega_{arm} = -1000 - 10\,\omega_{arm}$

$16\,\omega_{arm} = -1000$

$\boxed{\omega_{arm} = -62.5 \text{ rpm}}$

(CCW)



Cond^n	Arm 5	Gear 2 (60)	Gear 3 (40)	Gear 4 (100)
arm is fixed, gear 2 rotating with (+x) rpm	0	$+x$	$-x\left(\frac{60}{40}\right) = -\frac{3x}{2}$	$-\frac{3x}{2}\left(\frac{40}{100}\right) = -\frac{3x}{5}$
arm is rotating with +y rpm	y	$+x+y$	$y-\frac{3x}{2}$	$y-\frac{3x}{5}$

Gear 2 fixed

$N_2 = 0 \Rightarrow y + x = 0 \Rightarrow x = -y$

$N_4 = -100$ rpm (CCW)

$\Rightarrow y - \frac{3}{5}x = -100 \Rightarrow y + \frac{3}{5}y = -100$

$$\boxed{y = -62.5 \text{ rpm}}$$

Shortcut: $\frac{\omega_2}{\omega_3} = -\frac{\omega_2}{\omega_3} \cdot \frac{\omega_3}{\omega_4}$

$\frac{\omega_2 - \omega_{arm}}{\omega_4 - \omega_{arm}} = -\frac{T_3}{T_2} \cdot \frac{T_4}{T_3}$

CCW: $\frac{0 - \omega_{arm}}{-100 - \omega_{arm}} = -\frac{100}{60}$

$\frac{\omega_{arm}}{100 + \omega_{arm}} = -\frac{10}{6}$

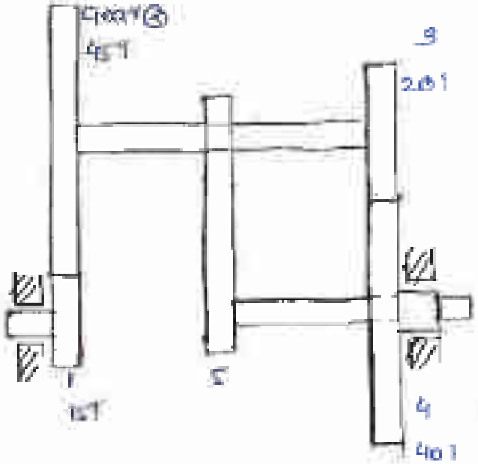
$\omega_{arm} = \ldots$

$$\frac{\omega_1}{\omega_4} = \frac{\omega_1}{\omega_2} \cdot \frac{\omega_2}{\omega_3} \cdot \frac{\omega_3}{\omega_4}$$

$$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{T_2}{T_1} \times \frac{T_4}{T_3}$$

$$= \frac{45}{15} \times \frac{40}{20}$$

$$\boxed{\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6}$$



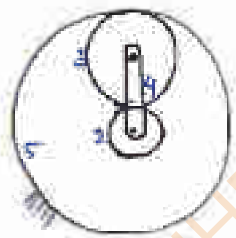
(b)
$$\frac{60 - \omega_5}{-120 - \omega_5} = 6$$

$\boxed{\omega_5 = -156 \text{ RPM}}$ (CCW)

[17]
$$\frac{\omega_2}{\omega_5} = -\frac{\omega_2}{\omega_3} \cdot \frac{\omega_3}{\omega_5}$$

$$\frac{\omega_2 - \omega_4}{\omega_5 - \omega_4} = -\frac{T_3}{T_2} \cdot \frac{T_5}{T_3}$$

$$\frac{\omega_2 - \omega_4}{0 - \omega_4} = -\frac{T_5}{T_2} = \frac{100}{20}$$



$-60 \;\; (\omega_2) - \omega_4 = 5\omega_4$

$6\omega_4 = -60 \Rightarrow \boxed{\omega_4 = -12}$ (CCW)

18]

Cond^n	arm	20 Gear 2	24 Gear 3, 32 Gear 4	80 Gear 5
arm is fixed, gear 2 rotates with +x rad/s	0	+x	$-x\left(\frac{20}{24}\right) = -\frac{5x}{6}$	$-\frac{5x}{4}\left(\frac{32}{80}\right) = -\frac{x}{3}$
arm is rotating with +y rad/s	+y	y + x	$y - \frac{5x}{6}$	$y - \frac{x}{3}$

on $\omega_{shaft} = 50$ rad/s (ccw)
$\omega_F = ?$

$y + x = 100$ —(1)

$y = -50$ (ccw)
$x = 150$

→ $\omega_F = y - x/3 = -50 - 150/3$

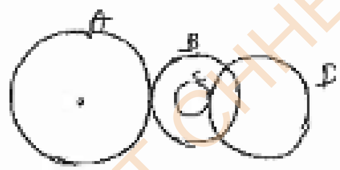
$\boxed{\omega_F = -100}$ (ccw)

Ex [19] → If engine shaft is not transferring any power at that time both the wheel rotate in opposite direction. If engine shaft is supplying power they will rotate in same direction due to differential gear box.

→ Differential gear box is used to provide different velocities to the inner & outer wheel while taking a turn because both the wheel has to cover different distance.

Ex [20]

$T_A = 50$
$T_B = 25$
$\omega_B = 100$ rpm
$\omega_A = ?$, $T_D = ?$

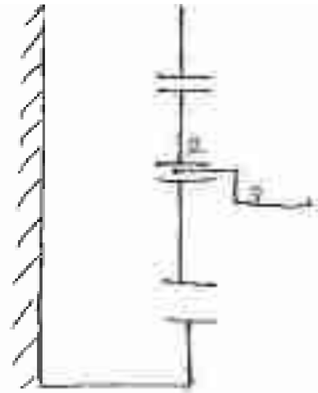


→ $\frac{\omega_A}{\omega_D} = \frac{\omega_A}{\omega_B} \cdot \frac{\omega_B}{\omega_C} \cdot \frac{\omega_C}{\omega_D}$

$\frac{\omega_A}{100} = \frac{T_B}{T_A} \times \frac{T_D}{T_C} = \frac{25}{50} \times \frac{T_D}{T_C} = \frac{25}{50} \times \frac{T_D}{50}$

$\frac{N_A}{100} = \frac{25}{50} \times \frac{T_D}{50}$ —(1)

→ $\boxed{N_A = T_D}$ ⇐ 50 rpm & 50 teeth



Cond^n	carrier 3	gear 2 96	gear 104
carrier is fixed gear 2 is rotating (+x) rpm	0	+x	$-x\frac{96}{104}$
carrier is rotating (+y) rpm	+y	x + y (find)	$y + x\frac{96}{104}$

$$y + x\frac{96}{104} = 0$$

$$x + y = 60$$

$$\frac{96}{104}x + y = 0$$

$$x + y = 60$$

$$\left(\frac{96}{104} - 1\right)x = -60$$

$$-\frac{8}{104}x = -60$$

$$\boxed{x = 104}$$

given $\boxed{y = 60 \text{ rpm}}$ (ccw)

$$y + x\left(\frac{96}{104}\right) = 0$$

$$(104)(60) + x(96) = 0$$

$\boxed{x = -65 \text{ rpm}}$

Gear 2 $= y + x = \underline{-5 \text{ rpm}}$ (cw)

fixing torque

The train is in equilibrium;

$$\sum T_{net} = 0$$

$$T_s + T_p + T_{arm} + T_A = 0$$

- Hence we can neglect the torque associated with planet.

$$T_s + T_{arm} + T_A = 0$$

- Since planet gear is neither connected to i/p nor to the output, hence we can neglect the torque associated with planet.
- There is no power in the system.

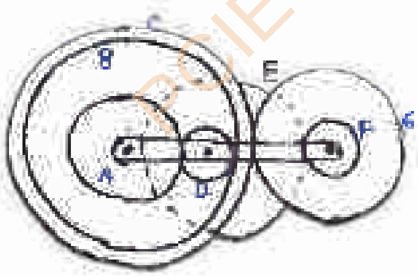
power @ o/p = power @ i/p

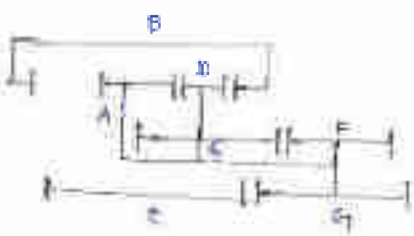
$$T_s\omega_s + T_{arm}\omega_{arm} = T_A\omega_A$$

- The torque associated with a gear is fixed is known as fixing torque.

Ex. In a gear train, gear DE and FG are compound gear. If the wheel A is fixed and the arm makes 90 revolution clockwise, find the revolution of B & C. If the arm is applied a turning moment of 1 kN m determine the turning moment on the shaft supporting the wheel C.

→ $T_A = 60$
$T_B = 120$
$T_C = 135$
$T_D = 30$
$T_E = 75$
$T_F = 30$
$T_G = 60$





Condition	Arm	60	20 / Gear C 75	120	30 / 60	135
Arm is fixed & gear A rotates with (x) rev	0	$+x$	$-x\left(\frac{60}{30}\right) = -2x$	$-2x\left(\frac{30}{120}\right) = -\frac{x}{2}$	$2x\left(\frac{75}{30}\right) = +5x$	$-5x\left(\frac{60}{135}\right) = -\frac{20x}{9}$
Arm is rotating y rev	y	$+x+y$	$-2x+y$	$-\frac{x}{2}+y$	$+5x+y$	$-\frac{20x}{9}+y$

A when fixed

$N_A = 0 \Rightarrow x + y = 0$

given $y = 20$ (given) $\boxed{x = -20}$

$N_E = -\frac{20}{9}(-20) + 20 \Rightarrow \boxed{N_E = 64.44 \text{ rev}}$

$\Rightarrow$ Equilm eqn $\Sigma T_{net} = 0$

$T_{arm} + T_A + T_B = 0$

The net power loss = 0

power @ i/p = power @ o/p

$T_{arm}\,\omega_{arm} + T_A\,\omega_A = T_E\,\omega_E$

since gear A is fixed $\boxed{\omega_A = 0}$

$T_{arm}\,\omega_{arm} = T_E\,\omega_E$

$1 \times 20 = T_E \times 64.44$

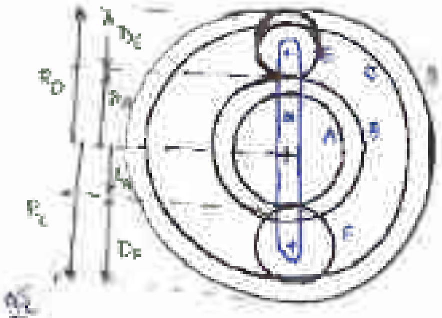
$\boxed{T_E = 0.310 \text{ kN m}}$

Holding torque

$1 + T_A + 0.310 = 0 \Rightarrow \boxed{T_A = -1.310 \text{ kN m}}$

speed of gear C, if the wheel D is fixed & arm rotating at 200 revolution clockwise

$T_A = 52$, $T_C, T_D = 1$
$T_B = 56$,
$T_E = T_F = 36$
Wheel D fixed
$N_{arm} = 200$ rev
$T_D = ?$



$\frac{\omega_A}{\omega_F} = \frac{\omega_A}{\omega_B} \times \frac{\omega_B}{\omega_E} \times \frac{\omega_E}{\omega_C} \times \frac{\omega_C}{\omega_D} \times \frac{\omega_D}{\omega_E}$

$\frac{56}{1} \frac{200}{N_B} = \frac{T_E}{T_B} = \frac{56}{56}$

$N_E = 311.11$ rev

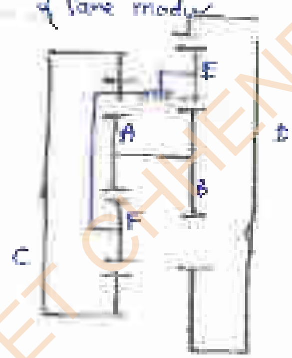
→ $R_D = D_E + R_A \Rightarrow$ $D_D = 2D_E + D_A$

$\frac{D_D}{m} = \frac{2D_E}{m} + \frac{D_A}{m}$ {same module

$T_D = 2T_E + T_A$

$T_D = 2(36) + 52$

$T_D = 124$



→ $R_A + D_F = R_C \Rightarrow$ $\frac{D_A}{m} + \frac{2D_F}{m} = \frac{D_C}{m}$

$T_A + 2T_F = T_C$

$52 + 2(36) = T_C$

$T_C = 124$

cond"	Arm	Gear A/Gear B 52 / 56	Gear F 36	Gear C 124	Gear E 36	Gear D 124
	0	$+x$	$-x\left(\frac{52}{36}\right)$	$-\frac{52}{36}x \cdot \left(\frac{36}{124}\right) = -\frac{52x}{124}$	$-x\left(\frac{56}{36}\right)$	$-x\left(\frac{56}{36}\right)\left(\frac{36}{124}\right) = -x\frac{56}{124}$
	y	$x+y$	$y - x\left(\frac{52}{36}\right)$	$y - \frac{52}{124}x$	$y - \frac{56}{36}x$	$y - x\left(\frac{56}{124}\right)$

$y = 7100$ ...

$x = 457.14$ ...

$N_c = y - \frac{x}{726} \times 58$

$N_c = 8.27$ ...

(3) Fig (a) OA = 640 mm

Fig (b) EA = 450 mm, FB = 160 mm, angle $\theta = 30°$ were co... Show that speed of rotation is same in both cases. Calculate % change in speed for 50 mm rise in the level of governor balls